An Artificial Stock Market

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ABSTRACT
In this paper, we present a model that simulates the behaviour of a heterogenous collection of financial traders on a market. Each trader is modelled as an autonomous, interactive agent and the agregation of their behavior results in market behaviour. [1] We specifically look at the role of information arriving at the market and the influence of heterogeneity on market dynamics. The main conclusions are that the quality of the information determines how the market will behave and secondly, heterogeneity is required in order to find the right statistical properties of the price and return time series.

KEY WORDS
Stockmarket, Agent based simulation

1 Introduction

In this paper, we present a model that simulates the behaviour of a heterogenous collection of financial traders on a market. Each trader is modelled as an autonomous, interactive agent and the agregation of their behavior results in market behaviour. We emphasize that the main goal of the paper is not to predict the future evolution of any stock, but rather to gain a deeper understanding of the phenomena observed in financial markets.
The main contributions of the paper are the following :

- The simulations suggest that the information arriving at the market determines to a high degree how the market will behave.
- In function of the information arriving at the market, crashes or speculative bubbles appear.
- Furthermore, it appears that in introducing heterogeneity, the overall market dynamics changes. An even stronger claim is that only by introducing heterogeneity does the model reproduce a market dynamics similar to real world financial price dynamics.

The paper is organized as follows. We first introduce the model and explain how each agent is modelled and how their interaction results in the overall market behavior. We then present the results of three simulation runs, one for a crash situation, one in which a speculative bubble appears and one representing a ’normal’ situation.

[1] The approach was first described in [7]. Another reference for the potential usefulness of the appraoch is [8].

2 Related Research

The Santa Fe Artificial Stock Market, as described in [1], served as a starting point for the model described in this paper. In their model, financial agents are recursive in nature as they form beliefs and expectations about the market on the basis of what they believe will be the other agents’ expectations. They claim to provide an answer to an old debate in which practitioners claim that there are speculative opportunities in the market, whereas academics believe in its efficiency. Their model shows that both views are correct, given the degree of explorative capabilities of the agents. In short, when agents are not, or just a little, allowed to explore alternative expectational models, the market price converges to the rational expectations equilibrium price. However, when agents can explore alternative models, a complex price pattern emerges allowing the emergence of bubbles or crashes.

Another model of a stock market using a similar approach is described in [2]. Each agent is described by a mathematical function and he uses a set of rules to form expectations about the future prices of a stock. This approach is different from the one used in [1], or in this paper, as the learning is implemented as a modification of the parameters of the mathematical function describing each agent. The main findings are that the initial wealth held by an agent and the method used to predict future prices largely influence the success of that agent on the market.

In [3], volatility clustering is explained in terms of certain proportion between chartist and fundamentalist trading strategies present on the market. Their model, using also the interactive agents approach, shows that when a certain threshold is surpassed, a sudden outbreak of volatility occurs. They see similarities with the on-off intermittency behavior in physics. They furthermore verify that the artificially generated time series have the same properties as real world financial data.

3 Description of the Model

3.1 The Agent’s Behaviour

We distinguish between different kinds of traders on the market, each having their own rationality and knowledge. As any financial trader, the agent must be able to evaluate an action and form an expectation with respect to its future price. On the basis of this expectation, he will propose a price to sell or buy a particular stock. This offer can then be evaluated by other traders on the market. These expectations are the result of some kind of reasoning and decision making. Depending on the success of the proposed transaction, measured in terms of financial profit, he will modify his decision rules and thus learn.

3.1.1 Model of an agent's behaviour

Decision making and expectations formation : as explained above, each agent needs to be able to decide whether he wants to buy or sell a particular stock, and at what price. He therefore needs to have decision rules that allow him to make some kind of expectation as to the future evolution of the price. He will do so on the basis of information at his disposal. In our model, we have chosen to implement a classifier system where different decision rules are represented as if-then rules. At a given moment, if a condition of his set satisfies the present situation in the environment, the agent will take the corresponding action. The condition of each rule is a chain of characters(" 0 ", " 1 ", or " # ") determining whether the rule is equivalent to the market situation. This equivalence is achieved if the characters along the chain of the condition are similar to the characters along the chain of the market situation. In the case of character " # ", there is always equivalence to the extent that it expresses the indifference between the characters " 1 " and " 0 ". As for the action, it is a chain of characters representing the value of two parameters *a* and *b* in binary fashion. These parameters allow to compute the expected future prices and dividends in the following way : $E[P_{t+1} + d_{t+1}] = a(P_t + d_t) + b$. For each agent, a set of rules allowing to calculate these expected prices and dividends will be generated using genetic algorithms. Initially, 900 rules are generated. This number will be reduced during the learning process.

Learning : in this original set of 900 rules, some may be more efficient than others. Those rules yielding more accurate expected prices and therefore a higher financial gain will have a higher reproduction rate and a higher probability to survive. The frequency of the reactualization of the rules will depend on each agent's ability to learn.

3.2 Model of the market

Information : as in real life, expectations with respect to prices and dividends are largely influenced by information arriving on the market. In our model, information arrives at the market at regular intervals of time. This information may vary from 'very negative (-3)' over 'neutral (0)' to 'very positive (+3)'. Figure 1 represents the distribution of the different kinds of news flashes for a normal situation. We emphasize that not every agent may interpret the same piece of information in the same way.

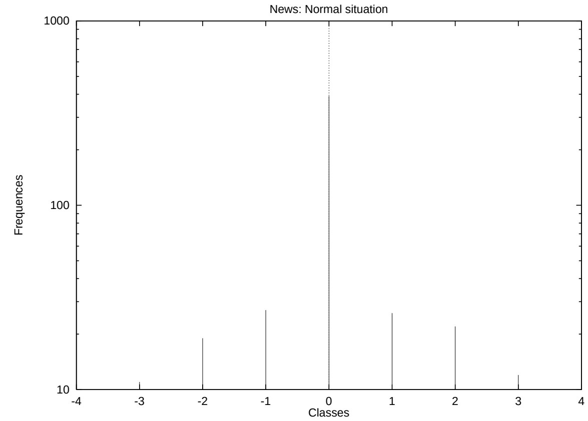

Figure 1. The Information Frequency Distribution : Normal Situation

Price formation and market clearing : Intersecting orders to buy and sell are going to create the dynamics of asset prices (see Figure 2). The market clearing mechanism is similar to the one used in [1] in which bids are continuously resubmitted until a price is formed that clears the market. At each period of time, the agents try to optimize the allocation of risky and non-risky assets. Initially, the price and dividend previsions made by agent i at time t are normally distributed with an average of $E_{i,t}[p_{t+1} + d_{t+1}]$ and a variance $\sigma^2_{t,i,p+d}$. Demand (or supply) by agent i at time t is given by :

$$x_{i,t} = \frac{E_{i,t}(p_{t+1} + d_{t+1} - (1+r)p_t)}{\lambda \sigma^2_{i,t,p+d}} \tag{1}$$

where p_t is the price of the asset at time t and λ is the degree of risk aversion.

In order to close the system, total demand must be equal to the number of available goods on the market :

$$\sum_{i=1}^{N} x_{i,t} = N \tag{2}$$

4 Simulation Results

In this section, we start by defining the general approach used. Before starting the actual simulations, we have allowed each agent to modify his decision rules on the basis of an artificially generated set of data. The goal of this "mode setting" is twofold : in that way, we can already reduce the 900 rules to a more manageable couple of hundreds. And secondly, most of the rules obtained as the result of the learning process, will make more sense than the original ones who were generated randomly.

It is important to emphasize that we are currently not looking at real world markets. The main reason is the lack

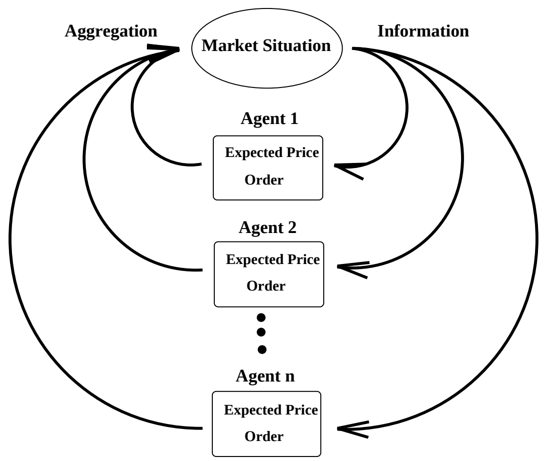

Figure 2. Aggregated Market Behaviour

of good empirical data in which the price evolution of any financial asset is linked to information arriving on the market (such as news flashes from Bloomberg or Reuters). We therefore impose a particular market behaviour. This means that we generate a time series, representing the price evolution in function of a particular information vector.
To this purpose, an artificial time series is generated, using the following equation :

$$P_t = (1 + \alpha I_{t-1})P_{t-1} \quad (3)$$

where I_{t-1} represents the information and P is the price. Once the agents have learned this mechanism the actual simulations can start. In the remainder of the paper we will use the following terms :

- Normal Agents : are those agents that will have learned this mechanism of how to use the information to compute a future price.
- Perturbating Agents : are those agents who will deviate from this mechanism.
- Reference Time Series : this is a time series computed using equation 3 on the basis of a new information vector.
- Generated Time Series : these are the ones generated by the interacting agents.

For each of the simulations, we compute the following statistics and which are summarized in Tables 1 to 4 :

- correlation coefficient between the reference time series of the prices and the generated one.
- The standard deviation measures price volatility.
- Skewness and Kurtosis are computed on the returns. A positive skewness and positive excess kurtosis are characteristic for real world financial data. [2]

[2] For a justification of these statistics we refer to [6] and [5].

Sim.–RAT	10	7	5	4	3	2
Normal	0.94	0.96	0.91	0.88	0.32	0.14
Crash	0.99	0.95	0.92	0.88	0.72	0.70
Bubble	0.99	0.99	0.98	0.98	0.96	0.96

Table 1. Correlation Coefficient

Sim–RAT.	10	7	5	4	3	2
Normal	1217	894	587	519	234	258
Crash	4561	2505	1598	1216	362	356
Bubble	5120	3638	2813	2641	1733	1254

Table 2. Volatility (Standard Deviation)

Sim.–RAT	10	7	5	4	3	2
Normal	0.07	-0.43	-0.29	-0.21	0.65	0.70
Crash	-0.99	-1.22	-0.89	-0.48	0.28	0.29
Bubble	1.26	1.62	2.83	3.36	4.53	4.11

Table 3. Skewness

Sim.-RAT	10	7	5	4	3	2
Normal	1.68	2.01	4.09	4.48	14.89	14.50
Crash	1.01	2.79	4.46	6.04	13.33	12.69
Bubble	4.17	5.85	13.19	16.26	23.39	24.08

Table 4. Excess Kurtosis

Type	7 RAT	5 RAT	4 RAT	3 RAT	2 RAT
inverse	1	2	2	3	3
Filter	1	2	3	3	3
Crazy	1	1	1	1	2

Table 5. Introducing Heterogeneity in the Simulations

For each set of simulations, we introduce heterogeneity by giving, in consecutive runs, a certain number of agents different kinds of decision making behavior. Besides the agents that will use Equation 3, we will have crazy agents who behave totally random, agents who will always the opposite of the 'normal' ones (inverse) and the third category of heterogeneous agents are the ones that attach less importance to extreme values of the information arriving at the market (filter). Table 5 summarizes the proportion of each type for each of the runs.

5 Normal Regime

We first discuss the results of the different simulations using a normal distribution of the news flashes arriving at the market. In three consecutive simulations, we increased the number of deviating agents.

Simulation 1

In this simulation, we modelled 10 agents having different sets of parameters (a and b) and each having his learned set of decision rules. The information vector used for this simulation is different than the one used during initial learning. If the agents have learned well the price dynamics during the initial learning phase, we expect that they should be able to reproduce similar (but differerent) dynamics. The differences could then be primarily due to the differences in the exogeneous variable I_t. The price dynamics is given in Figure 3. The correlation coefficient between the reference time series and the generated time series is 0.95 which shows a great similarity between the two. The skewness is negative but very small, indicating that the distribution of the returns is quasi normal. A positive excess kurtosis implies that the distribution is peaked. This seems to imply the following :

- the agents reproduce the correct dynamics. This claim is supported by the correlation coefficient of 0.95.
- The interaction on the market does not introduce a higher (positive) skewness even though the returns have a peaked distribution.
- We might also advance that the agents are apparently applying the decision mechanism they were taught.

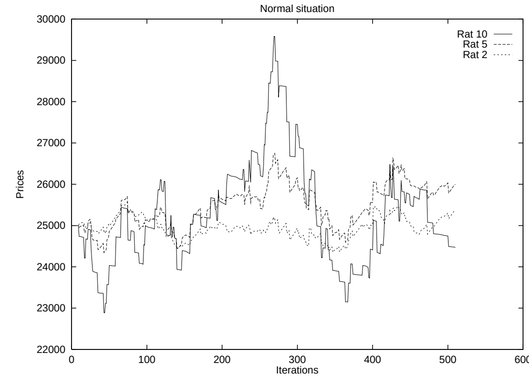

Figure 3. Price Dynamics of Simulations 1 and 2

Simulation 2

We now investigate whether or not the presence of perturbating agents can influence the market in such a way that the dynamics change. This boils down to introducing heterogeneity in the agents. To this purpose, we introduce, in consecutive simulations, from one to 8 perturbating agents that will systematically react differently than the others. Their interpretation of the information arriving on the market will be different, pushing them to make a different decision. The simulation counts the same number of periods and the same information vector has been used. This way, we can better compare the resulting prices with the time series of the previous simulation. The correlation coefficient goes down from 0.9 to 0.14, as the number of perturbating agents increases. Volatility decreases from 1217 to 258, as measured in terms of standard deviation. We also observe the occurence of a fat tail in the returns (skewness of 0.266) as more perturbating agents are introduced. Several similar runs of the model seem to indicate that due to differences in initial states, the proportion at which a positive skewness occurs differs. In all cases, we have observed an excess kurtosis. the market does not seem to be reacting in a systematic way to news flashes.

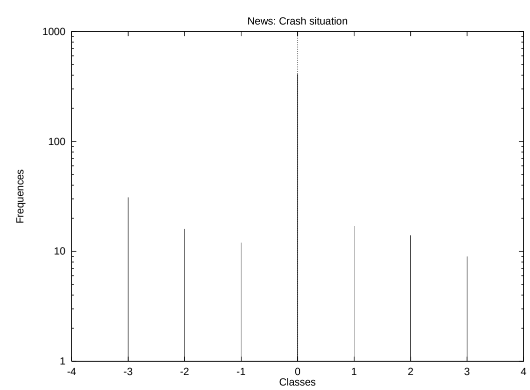

Figure 4. The Information Frequency Distribution : Crash

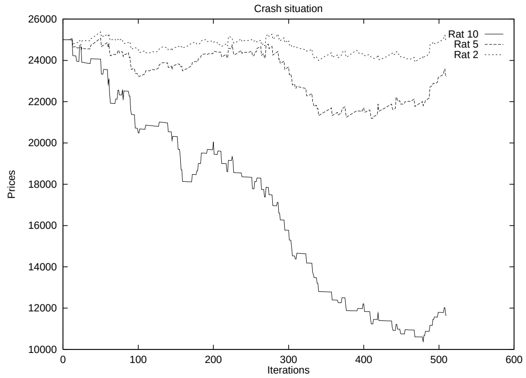

Figure 5. Price Dynamics of Simulation 3 and 4

6 Market Crash

Simulation 3

We now introduce a new information vector as the basis for the market dynamics. Rather than looking at a normal market situation, where there is no dominant trend in the information, we now simulate the situation in which bad news arrives at the market in a more or less constant way. The distribution of the information is given in Figure 4. As we can observe from Figure 5, there is a clear negative trend in the market. We also see from the computed standard deviations, that the volatility has increased drastically (from 1217 to 4755) which is in concordance with reality. Markets in crisis behave always more nervously than markets in a normal state. We also see that the correlation coefficient is still very high.[3] This leads us to suppose that still the same underlying decision taking mechanism is applied. However, skewness and kurtosis have dropped again to negative values.

Simulation 4

We again introduce, in consecutive runs, a number of perturbating agents. However, these agentsare different than the perturbating ones in simulation 2. The heterogeneity is introduced by imposing these agents to attach less importance to very negative information. The information arriving at the market is the same as in the previous simulation. As we can see from Figure 5 and from Tables 1 and 2, the market trend is still downward in all cases even though the downward trend diminishes as the number of perturbating agents increases. This is logical given the 'rationality' imposed on the perturbating agents. The correlation coefficients remain high (from 0.99 to 0.71) and the volatility diminishes as the number of perturbating agents increases. The volatility remains systematically higher than the volatility in the normal regime. Again, a positive skewness occurs when more heterogeneity is introduced and in all of the simulations, an excess kurtosis is found. The following conclusions can be advanced :

- Analogously to simulation 2, we observe that in introducing heterogeneity in the agents, the generated time series have properties similar to those of real world financial data.
- We furthermore see that the constant inflow of bad news, causes the market to crash. The price dropped 50% and volatility, compared to the volatility of simulation 2, has risen with a factor of 4.

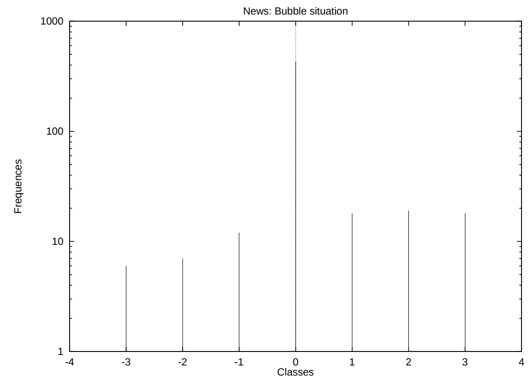

Figure 6. The Information Frequency Distribution : Bubble

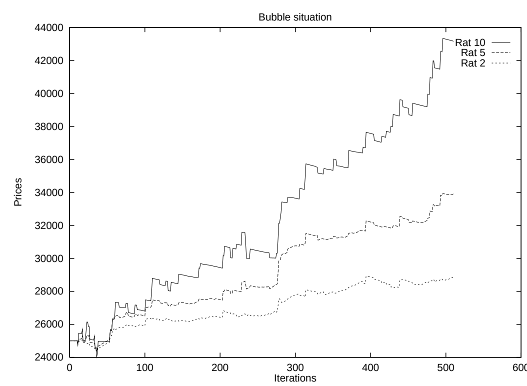

Figure 7. Price Dynamics of Simulation 5 and 6

7 Speculative Bubble

Simulation 5

In a third series of simulations, we introduce a new information vector (see Figure 6 where there is systematically good news arriving at the market). The price evolution is given in Figure 7. The first run in which all agents are similar, we see a clear upward trend of the market. The volatility is very high, compared to the normal regime, and we

[3] We now use as reference time series one that uses the same 'bad news' information vector as a point of comparison.

immediately have a positive skewness and an excess kurtosis.

Simulation 6

We again introduce heterogeneity in consecutive runs to see how the market behaves. Again, we observe that the bubble is less pronounced and even seems to disappear when heterogeneity is increased. The correlation coefficient remains high (from 0.99 to 0.96). Volatility goes down from 5120 to 1254 but, similar to the crash situation, remains systematically higher than the volatility in the normal case. Skewness increases with the number of heterogeneous agents, just as the excess kurtosis increases from 4 to 23.

8 Conclusion and Further Research

In this paper, we have presented some preliminary results of the simulations with an artificial financial market. The main conclusions are that information plays a crucial role in the way the market behaves. Each set of simulations clearly shows a different behavior of the market when different information sets are used. The second main conclusion is that only when introducing heterogeneity amongst the agents, does the model generate a market dynamics which exhibits similar characteristics as real world financial markets. A third conclusion that can be drawn from the above results is that, as far as the volatility is concerned, the increase of this statistic is observed whenever the market tends towards a crash or a speculative bubble. Further research is needed to confirm the above results. One of the things one might look at is what the influence is of different proportions of normal and perturbating agents.

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