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Degradation Stochastic Resonance (DSR) in AD-AVG Architectures

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Abstract— **This paper introduces for the first time the Degradation Stochastic Resonance (DRS) effect observed in the Adaptive Averaging (AD-AVG) architecture. This phenomenon, closely related to the well-known Suprathreshold Stochastic Resonance (SSR), influences the AD-AVG behavior for specific noise conditions and causes a yield improving effect over the degradation in time. In this article we analyze this counter-intuitive effect and explain the most relevant features. We observe, for example, that the yield of 20-input AD-AVG with 0.4 V of noise in the variability monitor increases from 0.93 to 0.97 after particular amounts of degradation.**

I. INTRODUCTION

New emerging device technologies may suffer from a reduced device quality. Along with the benefits of smaller size, low power consumption and high performance, future technologies are expected to have associated higher levels of process and environmental variations as well as performance degradation due to the high stress of materials [1], [2], [3], [4]. The development of fault-tolerant architectures emerges as a key research topic at the present. Currently, most of the fault-tolerant techniques rely on the use of majority gates [5], [6]. A successful alternative to majority gates is the averaging cell (AVG) [7], [8], [9], which exhibits higher reliability at lower cost based on the average of the input replicas. This approach is quite effective in case the inputs are subject to independent drifts with similar magnitude. However, this condition is no longer valid for the current technologies. To optimize the AVG architecture in heterogeneous drift environments we proposed the Adaptive Average cell (AD-AVG) [10]. This enhanced AVG technique is capable of tolerating non homogeneous input drifts and the effects of degradation by cleverly adapting the input weight values.

The AD-AVG has been analyzed in detail in previous papers [10], but recently it has been discovered an interesting but not so intuitive effect that deserves a specific justification. In this paper we describe for the first time a stochastic resonance phenomenon discovered in the behavior of the AD-AVG at specific conditions. This counter-intuitive effect takes places in the AVG structure as result of the influence of degradation in the hardware combined with the noise affecting the variability monitor used to reconfigure the averaging weights. The so-called Degradation Stochastic Resonance (DSR) effect is related to the well-known Suprathreshold Stochastic Resonance (SSR), which was first analyzed by Stocks in 2000 [11]. DSR basically involves an enhancement in the behavior of a signal processing system (AD-AVG) subject to noise and degradation. In the following sections we present the AD-AVG structure, the statistical models used to analyze its behavior and demonstrate the impact of DSR effect in the reliability of this fault-tolerant architecture.

II. THE ADAPTIVE AVERAGING CELL (AD-AVG)

The AD-AVG architecture, graphically depicted in Figure 1, is a fault-tolerant technique based on hardware redundancy. It calculates the most probable value of a binary variable from a set of error-prone physical replicas. The AD-AVG is demonstrated to tolerate high amounts of heterogeneous variability and accumulated degradation in the physical replicas [10].

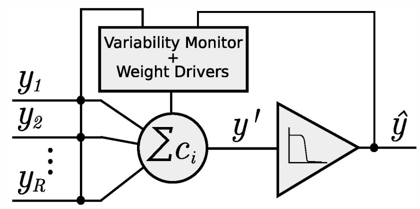

Fig. 1. Adaptive Averaging cell (AD-AVG) architecture.

The AD-AVG operation is based on a weighted average of R input replicas y_i of a binary variable y.

$$y_i = y + \eta_i \qquad i = 1, \ldots, R \tag{1}$$

Each replica y_i is affected by an independent drift η_i that alters the ideal value y. The y_i signals are represented in the system by continuous voltage levels, where 0 and V_{cc} stand for ideal logical values '0' and '1', respectively. Without loss of generality we use $V_{cc} = 1\ V$. The AD-AVG output $\hat{y}$ is an estimation of y according to (2).

$$y' = \sum_{i=1}^{R} c_i y_i \qquad \hat{y} = \begin{cases} V_{cc} & \text{if } y' > V_{cc}/2 \\ 0 & \text{if } y' < V_{cc}/2 \end{cases} \tag{2}$$

We model drift magnitudes as Gaussian random variables with null mean and different standard deviation levels $\eta_i \sim N(0, \sigma_i)$. The averaging weights c_i are normalized to the unity, i.e., $\sum_{i=1}^{R} c_i = 1$, and their optimal values for the reliability purpose are:

$$c_i = \frac{\sigma^2_{y'\ \min}}{\sigma_i^2} \qquad i = 1, \ldots, R. \tag{3}$$

Where $\sigma^2_{y'\ \min}$ corresponds to the minimum achievable weighted average variance. The input variances σ_i^2 are estimated by means of a variability monitor that is subject to noise. We refer to the level of noise in the monitor with the parameter σ_s. The averaging weights are reconfigured according to the input variance estimators and the optimal averaging weight values. A detailed presentation and discussion about the optimal averaging weights' formula can be found in [10].

In this work, we define the AD-AVG yield as the percentage of circuits that satisfies the reliability requirement $P_e < 10^{-4}$ (equivalently $\sigma_{y'} < \sigma_{\max} = 0.1344\ V$). Besides, in line with our previous AD-AVG related research [10] we use a degradation model to simulate the behavior of the system. We generate the initial values of the input replicas variance following a Gamma probability distribution function $\sigma_i^2 \sim \Gamma(x; k, \phi)$ with scale parameter $\phi = 2$ and mean value $E\{\sigma_i^2\} = 0.07\ V^2$ (this value reproduces technologies with poor reliability). The influence of increasing amounts of degradation is modeled by adding random Gamma-distributed increments to the initial input variances. We define the unit of degradation in time so that it corresponds to a mean increase in the replicas' variance σ_i^2 of magnitude $0.02\ V^2$. We use this normalized time-scale because the exact relationship between degradation and time is too complicated and depends on the particular technology used, the stress applied to the system and other environmental conditions.

III. Degradation Stochastic Resonance (DSR)

In this section we analyze the DSR effect in the AD-AVG architecture. To do so we first focus on a particular case of DSR with a 2-input AD-AVG structure. This example allows us to elucidate some circumstances that make the hardware degradation beneficial or detrimental to the AD-AVG reliability. After this example we also perform a sensitivity analysis of the AD-AVG yield to the degradation of one particular input. This result allows us to generalize the DSR effect to AD-AVG structures with an arbitrary number of inputs. Finally, we simulate AD-AVG structures with different redundancy factors R and noise levels in the variability monitor (σ_s) in order to give a clear overview of what implies the DSR effect in possible AD-AVG structures.

A. DSR in 2-input AD-AVG

Focusing on a simple AD-AVG case with 2-inputs we are able to perform an analytical study of the DSR effect. We analyze exactly what happens in a 2-input AD-AVG when one of the inputs has null, or very small, variability level ($\sigma_1 \approx 0\ V$) and the other one (σ_2) increases over time as a consequence of degradation. In order to perform the calculations we model the statistics of the input variability levels with parameters $\sigma_1 (= 0\ V)$ and σ_2, the noise in the variability monitor σ_s and the reliability specification of maximum admissible output variability $\sigma_{\max}$ (above this level we assume cell failure). First, we perform a Monte-Carlo simulation; Figure 2 depicts the resulting yield against the variability in the input 2 (σ_2). We observe that the yield is always 1 when $\sigma_2 < \sigma_{\max}$. This is because, in these conditions, any possible combination of weights gives place to an output with a variability level lower than the maximum admissible. However, as soon as σ_2 becomes greater than $\sigma_{\max}$ the yield begins to decrease at different rates depending on the level of monitor noise. Obviously, the greater the noise the greater the yield loss. This detrimental effect of degradation, or increase in the variability, happens only during a certain range of input variability. After a critical

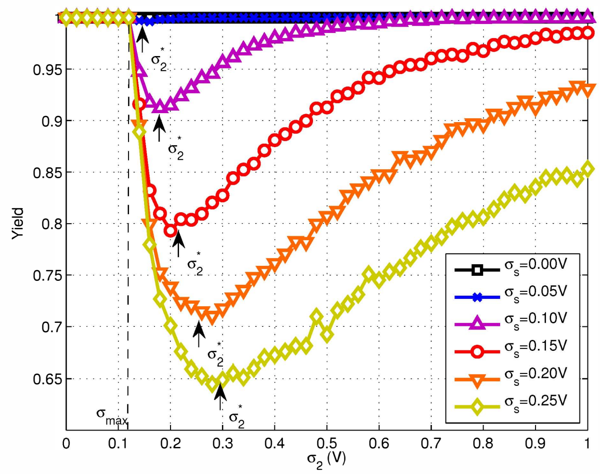

Fig. 2. Monte-Carlo simulation result of a 2-input AD-AVG yield against σ_2 with null variability in the input 1 ($\sigma_1 = 0\ V$). We consider different levels of noise in the variability monitor (σ_s) and a maximum admissible output variability of $\sigma_{\max} = 0.1344\ V$.

variability level in the input 2 (σ_2^*) the effect of degradation changes and becomes beneficial; this is the starting point for the DSR effect in this particular case. We can understand this phenomenon by noting that as the input variability σ_2 grows the AD-AVG is capable of calculating more precisely the optimal value of the averaging weights. The ratio between the variability σ_2 and the monitor noise σ_s increases and, therefore the measure of input variability needed to calculate the averaging weights becomes more reliable. In fact, the AD-AVG gives more weight to the input one, which has null variability, and the yield grows up to 1 again.

In this particular case we can prove this effect analytically. Given the conditions described for this example we can calculate the probability density function of the averaging weights $f_{c_1}(c_1)$, $f_{c_2}(c_2)$ and the yield of the A-AVG structure Y. Taking the derivative of the yield with respect to the variability level σ_2 we obtain a closed analytic expression that reveals the impact of degradation in the reliability of the structure, see Equation (4).

$$\begin{aligned}\frac{dY}{d\sigma_2} =& \frac{1}{\sqrt{2\pi}\sigma_s}\sqrt{\frac{\sigma_{\max}}{\sigma_2}}e^{-\frac{\sigma_{\max}\sigma_2}{2\sigma_s^2}}\operatorname{erf}\left(\frac{\sqrt{\sigma_2(\sigma_2-\sigma_{\max})}}{\sqrt{2}\sigma_s}\right) \\ &- \frac{\sigma_{\max}}{\pi\sigma_2\sqrt{\sigma_2(\sigma_2-\sigma_{\max})}}e^{-\frac{\sigma_2^2}{2\sigma_s^2}} \end{aligned} \quad (4)$$

Matching to zero Equation (4) we obtain the condition in terms of accumulated degradation (σ_2) that the AD-AVG system must satisfy in order to start experiencing the DSR effect, see Equation (5). In this example we define this characteristic point of change from detrimental to beneficial degradation as the critical variability level σ_2^*.

$$\frac{1}{\sqrt{\pi}a} \times e^{-a^2} = \operatorname{erf}(a) \quad (5)$$

Where

$$a^2 = \frac{\sigma_2^*(\sigma_2^* - \sigma_{\max})}{2\sigma_s^2}. \quad (6)$$

Solving this equation we get the expression of σ_2^*:

$$\sigma_2^* = \frac{\sigma_{\max}}{2} + \sqrt{\left(\frac{\sigma_{\max}}{2}\right)^2 + 0.769\ \sigma_s^2}. \quad (7)$$

Substituting this formula with the simulation parameters of Figure 2 we can verify the result, see Table I. Using this relation we can easily find the critical variability level σ_2^* of the AD-AVG for any given noise level and reliability requirement.

TABLE I

CRITICAL VARIABILITY LEVELS (σ_2^*) OF DSR EFFECT IN A PARTICULAR CASE OF 2-INPUT AD-AVG WITH NULL VARIABILITY IN THE FIRST INPUT.

σ_s (V)	σ_2^* (V)
0.00	0.1344
0.05	0.1474
0.10	0.1777
0.15	0.2149
0.20	0.2550
0.25	0.2965

B. AD-AVG yield sensitivity analysis

In order to extend the previous result to AD-AVG systems with arbitrary number of inputs we perform a sensitivity analysis of the AD-AVG's yield to the variability level of one particular input (the ith one). With this experiment we demonstrate the occurrence of the DSR effect in a general AD-AVG case. To do this we assume an R-input AD-AVG and analyze the weighted average variance against the contribution of the ith input. We separate the general output variance expression as follows:

$$\sigma_{y'}^2 = c_i^2\sigma_i^2 + \sum_{j=1, j\neq i}^{R} c_j^2\sigma_j^2. \quad (8)$$

We generate random sets of $R-1$ variance values σ_j^2, $j = 1, \ldots, R$, $j \neq i$, following the Gamma distribution function as in previous studies [10]. As for the ith variability (σ_i), we swept its value from 0 to 0.4 V in the Monte-Carlo simulations. In these conditions, we estimate the AD-AVG's yield taking into account different levels of noise in the variability monitor (σ_s). Figure 3 depicts the simulation results against the influence of σ_i.

We note in this experiment that ideally (when the monitor noise is null) the AD-AVG yield is 1 as long as $\sigma_i < \sigma_{\max}$, and after this value the yield decreases gradually to 0.93. This happens because the AD-AVG structure with null noise perfectly optimizes the reliability. Therefore, while the ith variability is lower than the reliability requirement $\sigma_{\max}$ the AD-AVG yield is 1, and after this value the yield decreases gradually to the maximum yield achievable with the remaining $R-1$ inputs. On the other hand, when the variability monitor is affected by noise the optimal weight values are

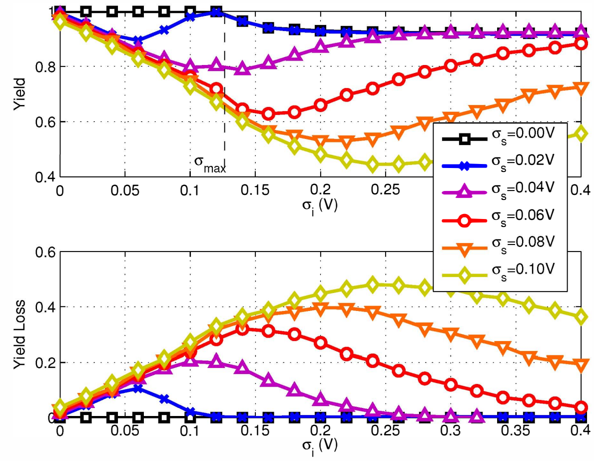

Fig. 3. Sensitivity analysis of AD-AVG's yield with multiple inputs to the variability of input ith for different levels of noise (σ_s) in the variability monitor.

calculated imperfectly and the yield is not maximized. We observe in the figure that the impact of this imperfect weight configuration is not homogenous throughout all the values of ith input variability. The yield loss presents a resonance effect with the ith input variability for different levels of monitor noise (σ_s). This result together with the previous example evidences the relevance of the DSR effect in the AD-AVG structure.

C. DSR in AD-AVG

Once we have perceived the intuition of the DSR effect and demonstrated its applicability to general AD-AVG cases we focus in this last subsection on the analysis of typical AD-AVG. Following with the previous simulations we analyze now AD-AVG structures against the degradation in time and observe the evolution of reliability. This time we need to use the degradation model described in Section II. We simulate different size AD-AVGs with different levels of noise in the variability monitor (σ_s) and estimate the corresponding yield. Figure 4 depicts the simulation results for redundancy factors $R = 3$, 10, and 20.

In the figure we clearly observe how the yield characteristic of AD-AVG changes over time due to the DSR effect. We also note that the influence of this resonance phenomena is more significative at higher levels of redundancy. In the examples of Figure 4 the DSR effect is not perceived for the 3-input AD-AVG whereas it does for the 10 and 20 inputs AD-AVG. Another conclusion that we can extract from the figure is that DSR implies a diminution of the negative influence of noise in the monitor when the accumulated level of degradation is significantly higher than the noise level. In fact, both Figure 3 and Figure 4 show a negative impact of noise in the monitor that increases for low levels of degradation or input variability and then decreases after a critical amount of degradation in time. We call this point the critical level of degradation in time.

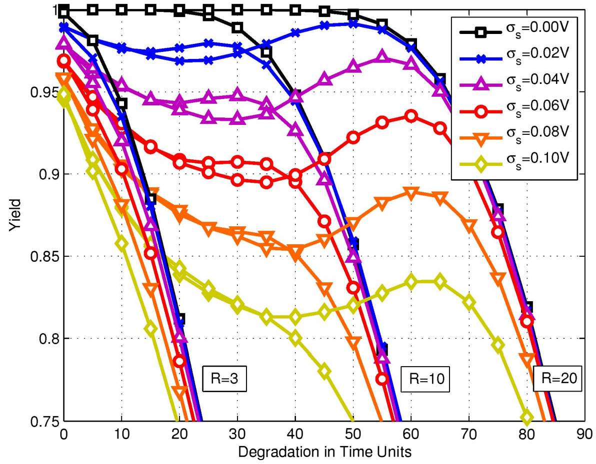

Fig. 4. Yield analysis of different size AD-AVGs against degradation for different levels of noise (σ_s) in the variability monitor.

Thanks to this effect it is possible to obtain higher factors of AD-AVG yield after specific amounts of degradation. Regarding this experiment on the DSR effect we can extract the following ideas:

- The DSR effect becomes more relevant in AD-AVGs with larger number of inputs.
- Given an AD-AVG in a particular situation of degradation in time and noise level it is not always the best option to use all the available replicas. There are situations in which less input replicas provide higher yield with the same degradation in time and noise in the variability monitor.
- It may be considered the option of adding a controllable noise generator to each of the input replicas to virtually increase the instantaneous amount of degradation in time up to the resonance point. This operation is feasible as long as the level of degradation in time is below the resonance point.

IV. Conclusions

In this article we present for the first time the Degradation Stochastic Resonance (DSR) effect found in the Adaptive Averaging (AD-AVG) structure. This counter-intuitive effect implies an enhancement in the system reliability against hardware degradation for specific noise conditions. We prove this behavior analytically in the particular case of 2-input AD-AVG with null variability in one of the inputs. The stochastic resonance effect is related to the ratio between the amount of degradation accumulated in the hardware and the noise level present in the variability monitor. The higher is this ratio the more accurate is the calculation of the optimum averaging weights. We perform several Monte-Carlo simulations to generalize the DSR effect implications to multiple input AD-AVGs. Exploring the main features of DSR we observe that this effect becomes more relevant in AD-AVGs with large number of inputs. Besides, there are particular circumstances in which it is better to use less replicas than the total available in order to maximize the yield. It also can be considered the option of adding a controllable noise generator to the inputs in order to virtually increase the amount of hardware degradation and shift the characteristic yield to the resonance point and maximize this way the yield.

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