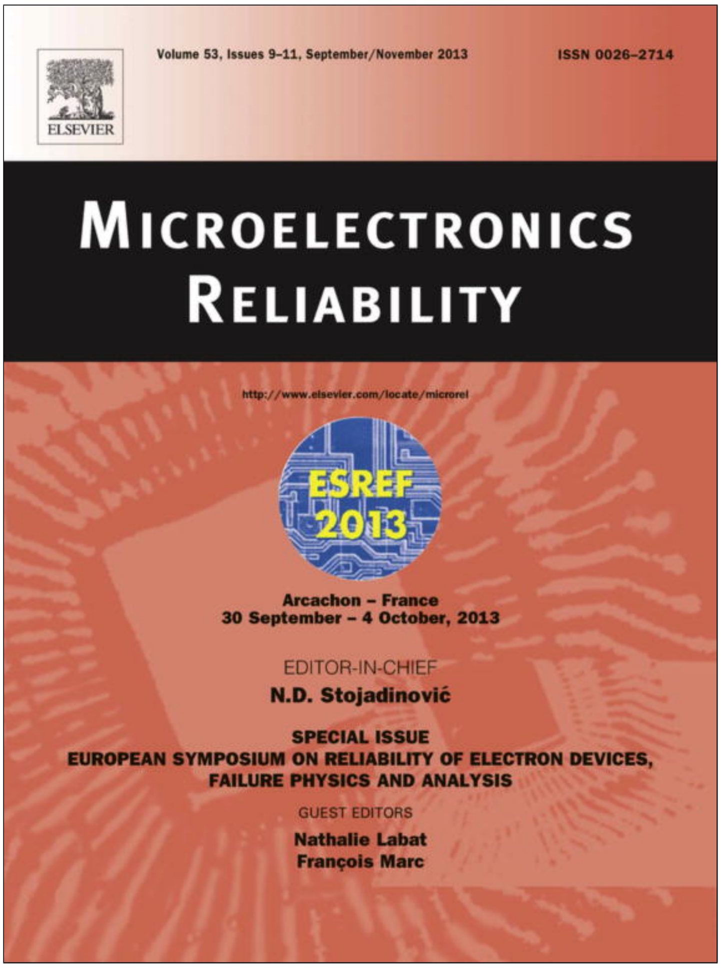
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A nonlinear degradation path dependent end-of-life estimation framework from noisy observations

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ABSTRACT

For the current advanced technology nodes, the end-of-life and reliability statistics estimation is regarded as a key component of devices dynamic reliability management frameworks. An accurate estimation can enable effective lifetime management via adopting appropriate mission profile specific policies. This paper proposes an end-of-life and reliability estimation framework, which takes into account the nonlinearities of the degradation process, as well as the sensors measurements and degradation process uncertainty, aiming to characterize more realistically the devices aging dynamics. Based on the degradation history, the estimation results are updated adaptively via the Bayesian method, once new degradation measurement data are provided. In order to validate and assess the estimation accuracy of the proposed framework, numerical simulations were performed on a power law degradation model. The obtained results for the considered nonlinear degradation process, reveal that, when compared with commonly employed Wiener processes with linear mean, our approach exhibits improved estimation accuracy. Thus, it may be better suited to capture the nonlinearity and variability of in-field degradation dynamics and further to assess/predict the devices reliability in a more realistic manner.

1. Introduction

The aggressive technology scaling for performance improvement has negatively impacted the devices lifetime reliability [1]. To address the aging-induced lifetime degradation of devices with minimal impact on the performance characteristics, Dynamic Reliability Management (DRM) frameworks have been developed. Since the effectiveness of the DRM policies depends on the reliability assessment accuracy, increasing attention has been payed to this topic. Most of past approaches [2–4] accept the modeling simplifying assumptions that the degradation process is monoton, and/or can be linearized using time-scale transformations, which can result in a conservative lifetime estimation. Only recently, degradation models that integrate a nonlinear structure to trace better the degradation dynamics have been proposed. In [5], the mean parameter of the degradation process was updated using Kalman filtering, but its uncertainty was not considered and the variance was assumed linear in time. In [6], the degradation nonlinearity was captured without data transformations; however only the current degradation data was used, disregarding the degradation history. The degradation history problem was addressed in [7], but the variance was also assumed linear as in [5].

In view of the above, we propose a Bayesian reliability assessment framework, which takes into account the nonlinearities of the degradation process, aiming to characterize more realistically the wearout process dynamics and thus to improve the potential effectiveness of adopted reliability management policies. This is achieved by using a Wiener process to govern the dynamics of the degradation process, with nonlinear mean, which is expressed as a combination of basis functions, weighted by degradation history dependent parameters. Based on the entire degradation history, and not only on the instantaneous degradation state, the degradation model parameters are updated via Bayesian inference, once new degradation data are accumulated. Furthermore, we account in the proposed reliability assessment framework for the uncertainty in both the degradation process and the measurements. Numerical simulations were carried out in order to validate and evaluate the estimation accuracy of the proposed approach in comparison with commonly employed Wiener processes with linear mean. The obtained results quantitatively confirm that, when compared to the linear mean Wiener process, the proposed framework may be better suited for capturing nonlinear in-field degradation dynamics and hence for assessing/predicting the reliability in a more realistic manner.

The remaining of the paper is organized as follows: Section 2 presents the degradation process formalism and the general modeling principles. The proposed framework is introduced in Section 3

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and validated and evaluated in Section 4 and 5. Section 6 concludes the paper with a summary of this work.

2. The degradation process formalism

Given that an Integrated Circuit (IC) is functional at the current time moment, based on its history of degradation (constituted by a set of noisy measurements collected from the in-field degradation sensors), one is interested in deriving its real-time reliability. According to the reliability status and the remaining operational life, appropriate lifetime management strategies can be adopted. Hence, the central problem of the IC reliability assessment/prediction, is inferring the End-of-Life (EOL) statistics.

A degradation (wearout, aging) process is stochastic in nature. One candidate stochastic process that can govern the dynamics of an IC wearout process is the Wiener process, denoted subsequently by $W(t)$. The degradation process can be governed by an equation of the following form:

$$dX(t) = \mu(\alpha, t)dt + \sqrt{\sigma}\, dW(t), \tag{1}$$

where $X(t)$ describes the degradation state at time moment t. The Wiener degradation process $W(t)$ is specified by its mean (drift) μ, and variance $\sqrt{\sigma}$, which describe the degradation evolution in time. The nonlinearity of the degradation process is captured in the nonlinear time variation of the functional μ, with the parameters vector α. In order to accommodate for the heterogeneity of an IC degradation sources during its lifetime, the drift μ can be regarded as being composed of two terms: (i) $g(x,t)$, which is a fixed, deterministic component, common to all ICs (e.g., measurement bias), and (ii) $\alpha \cdot f(x)$, which is a variable, a priori unknown nonlinear component, with $f(x)$, the set of basis functions (e.g., Gaussian, polynomial, fuzzy membership functions) and α, the unknown parameter vector. Consequently, (1) becomes:

$$dX(t) = g(x,t)dt + \alpha f(x,t)dt + \sqrt{\sigma}\, dW(t). \tag{2}$$

Therefore, the unknown parameters vector $\theta = (\alpha, \sigma)$ completely defines the degradation process, and has to be estimated from a set of noisy degradation measurements, $V(t)$. Having determined the IC degradation model, the future evolution of the degradation process can be predicted and the lifetime related properties of interest can thus be inferred. The general principle of the reliability estimation is graphically caught in Fig. 1.

Given a set of noisy degradation measurements V (e.g., degradation of an IC performance characteristic such as max. operating frequency), which constitute the degradation history up to current time moment t_k, the degradation process parameters θ are estimated. Based on the relation between a future degradation value and the up-to-date degradation history, given by the degradation process model, the potential future evolution paths of the degradation can be predicted. When a future degradation value exceeds a pre-specified threshold T (e.g., usually set to 10% degradation of the IC performance characteristic) for the first time, then the IC has reached its EOL. Hence, the EOL for a degradation path X can be defined as follows:

$$EOL = \inf\{t : X(t) \geqslant T | X(s) < T, 0 < s < t\}. \tag{3}$$

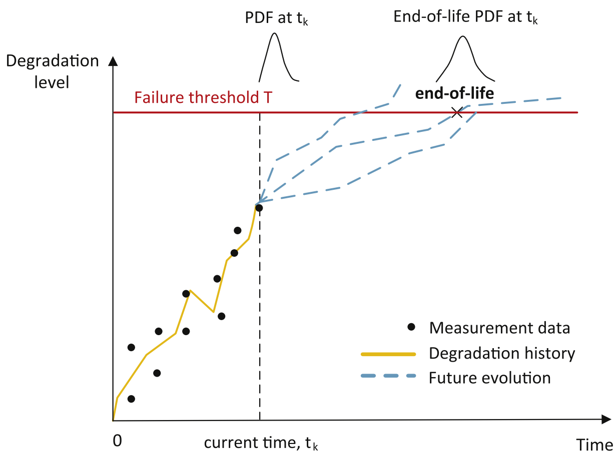

Fig. 1. Illustration of the reliability modeling principle.

The reliability at a time moment t for the ensemble of predicted degradation evolution paths, can then be obtained as the probability at time t of not reaching the EOL.

In view of the above, we shall present first the general methodology for deriving the device EOL in Section 3.1, followed by the corresponding algorithmic details in Sections 3.2 and 3.3.

3. The reliability assessment framework

For a given observation vector V, the parameters θ, which characterize the degradation process, are estimated taking into consideration the degradation history. The posterior distribution of the parameters θ is updated via a Bayesian framework [8], which enables to effectively integrate the historical, up-to-date degradation data together with the newly in situ degradation observations. Once θ and the degradation path are estimated, the EOL is given by the time moment when the degradation path exceeds the predefined threshold. By simulating an ensemble of degradation paths for the same θ, the reliability at a specific time can be derived as the probability of not exceeding the predefined threshold.

3.1. Reliability evaluation procedure outline

The joint posterior distribution of θ and $x_{1:N}$, conditional on the observations V, can be sampled without having to compute the density, by using the Gibbs sampling algorithm [9], which alternates between the following two steps, for M times:

1. draw $x_{1:N}|\theta$, V, i.e., generate a sample of the degradation path $x_{1:N}$, for fixed θ and given observations V, and
2. draw $\theta|x_{1:N}$, V., i.e., generate a sample of the parameters set $\theta = (\alpha, \sigma)$, for fixed degradation path $x_{1:N}$ and given observations V.

The latter step, i.e., the update of the $\theta = (\alpha, \sigma)$ parameters, is particularly straightforward, since conjugate prior distributions are employed for α and σ. This makes it possible to derive analytically the conditional distribution of θ, given the observations V and the degradation path $x_{1:N}$, and sample directly from it. The former step however, is more computationally demanding. In such case, to overcome the difficulties of direct sampling, a Metropolis Hastings numerical approach [8] is applied. Specifically, the mth iteration of the Gibbs sampler can be written as follows:

$$\text{sample } x_n^{(m)} \sim p\left(x_n|, \theta, V, x_{n-1}^{(m)}, x_{n+1}^{(m-1)}\right) \text{for } n = 1:N$$
$$\text{sample } \sigma^{(m)} \sim p\left(\sigma|x_{1:N}^{(m)}, \alpha^{(m-1)}\right)$$
$$alpha^{(m)} \sim p\left(\alpha|x_{1:N}^{(m)}, \sigma^{(m-1)}\right),$$

where $m = 1{:}M$.

For the given observation vector V, having generated M samples of θ from the updated posterior distribution of $\theta|x_{1:N}$, V, the EOL can now be inferred by simulating the M degradation paths using the discrete version of the continuous time dynamics governed by

(2). For this purpose, we employ the Euler–Maruyama approximation [10], with a discretization time step of resolution τ, as follows:

$$x^{(m)}_{N+(k+1)\tau} = x^{(m)}_{N+k\tau} + \tau \cdot g(x^{(m)}_{N+k\tau}) + \tau \cdot \alpha^{(m)} f(x^{(m)}_{N+k\tau}) + \sqrt{\tau} \cdot Z_{N+k\tau}, \quad (4)$$

where $Z_{N+k\tau} \sim \mathcal{N}(0, \sigma^{(m)})$. The number of discretization steps for each of the M paths, is determined by the EOL stopping criterium, i.e., when the degradation path sample $x^{(m)}_{N+(k+1)\tau}$ exceeds the EOL target (the threshold T, as defined in (3)). The EOL values for the M simulated paths, given the degradation history $x^{(m)}_{1:N}$ and the corresponding parameters set $\theta^{(m)}$ for each path, are computed as:

$$\mathbf{EOL}_{1:M} = inf\left\{EOL_m : x^{(m)}_{EOL_m} \geqslant T, \quad m = 1 \ldots M\right\}. \quad (5)$$

The reliability function at time instant t, can now be derived as:

$$R(t) = \{P(\mathbf{EOL}_{1:M} > t) : t > N\}. \quad (6)$$

With the above considerations in place, we are now in position to present the Gibbs sampler details for obtaining the parameters which characterize the degradation process.

3.2. Gibbs sampler step 1 – draw $x_{1:N}|\theta$, V

Given the parameters set $\theta = (\alpha, \sigma)$, the distribution of a degradation path $x_{1:N}$ can be obtained from:

$$p(x_{1:N}|\theta, V) \propto p(V \mid x_{1:N}) p(x_{1:N} \mid \theta), \quad (7)$$

where $p(x_{1:N}|\theta)$ is the probability of deriving the degradation path $x_{1:N}$ for the parameters set θ, and $p(V|x_{1:N})$ is the likelihood, the probability of observing the measured degradation path from $x_{1:N}$. The Markov property [8] of (4) implies that the conditional distribution of $x^{(m)}_n$, given all the other values is the same as the distribution given the adjacent endpoints $x^{(m)}_{n-1}$ and $x^{(m-1)}_{n+1}$. It follows that the posterior distributions of the parameters are:

$$p(x^{(m)}_n | x^{(m)}_1, \ldots, x^{(m)}_{n-1}, x^{(m-1)}_{n+1}, \ldots, x^{(m-1)}_N, \theta, V) \propto p(x^{(m)}_n | x^{(m)}_{n-1}, x^{(m-1)}_{n+1}, \theta, V),$$

which is further proportional to:

$$\propto p(V_n | x^{(m)}_n) p(x^{(m)}_{n-1} | x^{(m)}_n, \theta) p(x^{(m-1)}_{n+1} | x^{(m)}_n, \theta). \quad (8)$$

As concerns $p(V_n|x^{(m)}_n)$, it results from the distribution of the noisy observations, which is given a priori. Since $x_n|x_{n-1}$ in (4) follow a normal distribution, the probabilities $p(x^{(m)}_{n-1}|x^{(m)}_n, \theta)$ and $p(x^{(m-1)}_{n+1}|x^{(m)}_n, \theta)$ can be readily derived.

For the present purposes, in order to sample the target conditional distribution of $x_{1:n}|\theta$, V, we employ a numerical procedure, i.e., the Metropolis Hastings algorithm [8], outlined subsequently. Based on (4) with time step resolution h (which can be equal to the time sampling resolution of observed data), for existing x_n, a step x^*_n can be proposed by drawing from the distribution:

$$pdf(x^*_n|x_n) = \frac{1}{\sqrt{h\sigma 2\pi}} \cdot e^{-\frac{(x^*_n - x_n - hg(x_n) - h\alpha f(x_n))}{2h\sigma}}. \quad (9)$$

The acceptance probability of x^*_n as candidate to replace the current draw x_n, is given by:

$$\rho\left(x^{(m)}_n, x^{*(m)}_n\right) = \min\left\{\frac{q(x^*_n, x_n)}{q(x_n, x^*_n)}, 1\right\}, \quad (10)$$

where

$$q(x^*_n, x_n) = p(V_n|x^*_n) p(x_{n+1}|x^*_n) p(x^*_n|x_{n-1})$$
$$q(x_n, x^*_n) = p(V_n|x_n) p(x_{n+1}|x_n) p(x_n|x_{n-1}).$$

The conditional probabilities in (10) can be computed using (9).

3.3. Gibbs sampler step 2 – draw $\theta|x_{1:N}$

We are interested in drawing a sample of the parameters θ from the posterior probability distribution, which is given by:

$$p(\theta|x_{1:N}) \propto p(\theta) p(x_{1:N}|\theta). \quad (11)$$

To this end, we derive the likelihood of θ for a given degradation path $x_{1:N}$ as:

$$p(x_{1:N}|\theta) \propto \frac{1}{\sqrt{\sigma^N}} \cdot e^{-\frac{1}{2h\sigma} \sum_{n=1}^{N} \left[x_{n+1} - x_n - h \cdot g(x_n) - h \cdot \alpha f(x_n)\right]^2}. \quad (12)$$

For a successful Bayesian inference, we assume the prior distributions for the parameter set $\theta = (\alpha, \sigma)$ belong to the conjugate family [8] of the sampling distribution $p(x_{1:N}|\theta, V)$. As such, we consider an inverse Gamma distribution for the degradation process variance, i.e., $\sigma \sim \Gamma^{-1}(q_2, q_3)$, and a normal distribution for α, i.e., $\alpha \sim \mathcal{N}(0, q_1)$. Since

$p(\alpha|x_{1:N}, \sigma) \propto p(\alpha) p(x_{1:N}|\alpha, \sigma)$ for σ known
$p(\sigma|x_{1:N}, \alpha) \propto p(\sigma) p(x_{1:N}|\alpha, \sigma)$ for α known,

it follows that:

$$\begin{aligned}
&\alpha|x_{1:N}, \sigma \sim \mathcal{N}(\alpha_m, \alpha_v)\\
&\quad \alpha_m = -\alpha_v \cdot \sigma^{-1} \sum [x_{n+1} - x_n - h \cdot g(x_n)] \cdot f(x_n)\\
&\quad \alpha_v = \left[q_1^{-1} + \sigma^{-1} \cdot h \cdot \sum f^2(x_n)\right]^{-1}\\
&\sigma|x_{1:N}, \alpha \sim \Gamma^{-1}(\sigma_m, \sigma_v)\\
&\quad \sigma_m = q_3 + N\\
&\quad \sigma_v = q_2 + h^{-1} \sum [x_{n+1} - x_n - h \cdot g(x_n) - h \cdot \alpha f(x_n)]^2.
\end{aligned}$$

4. Performance evaluation

In order to validate the proposed approach, we consider the nonlinear process modeled by (2), with mean $\mu(\alpha, t)$ given by t^β ($g(x, t) = 0$). We conduct the numerical experiments employing the following parameters values: the number of degradation paths equal to 100, the discretization step $h = 0.1$, the Wiener process variance $\sqrt{\sigma} = 0.23$, and its mean power-law coefficient $\beta = 2$. As concerns the basis functions α modeling the process mean, without loss of generality, for simulation purposes, we employed the Gaussian kernel [11]. The estimation performance of the proposed model was studied using noisy observations sampled from $\mathcal{N}(x(t), 0.01)$. For estimation accuracy evaluation purpose, we compare with the commonly employed Wiener processes with linear mean given by $\alpha \cdot t$ [2–4].

In Fig. 2, the real degradation path generated via (4), is illustrated against the two degradation paths, estimated with the proposed nonlinear degradation model and with the reference linear model, respectively. In direct relation to ICs aging, the degradation path could represent the threshold voltage degradation of a transistor, the maximum operating frequency degradation of circuit, etc. It can be observed that, the proposed nonlinear degradation model exhibits a fairly well estimation ability, the real and estimated degradation paths almost overlapping. Furthermore, the fitting characteristics are improved when compared to the reference model with linear drift, for the considered degradation process. We note that while the estimation tends to be less accurate during the early period, characterized by a limited degradation history, the fitting characteristics improve in time, as degradation data are accumulated. The proposed model however yields a more accurate estimation than the model with linear mean, also during the early period with few degradation measurement data. We evaluated the

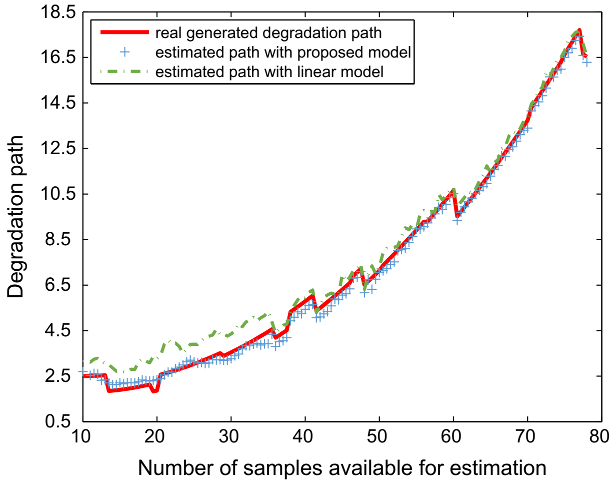

Fig. 2. The real vs. estimated degradation paths.

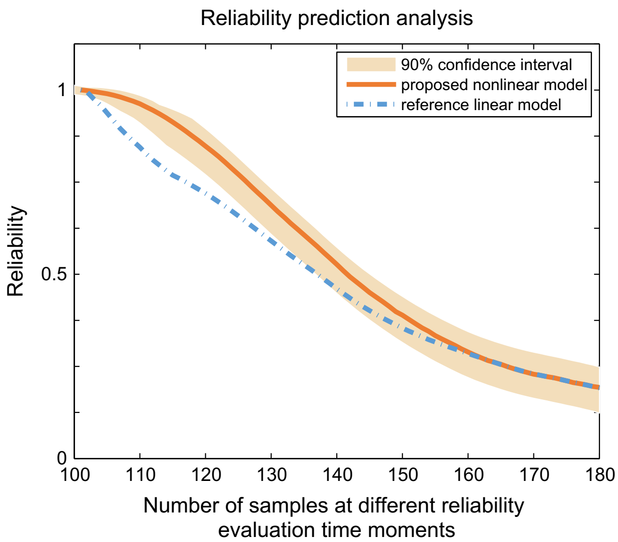

Fig. 3. The reliability assessment/prediction error analysis.

degradation path estimation accuracy using the Akaike Information Criterium [12], whose statistics is given by:

$$AIC = 2 \cdot k - 2 \cdot \max[\log(likelihood)], \tag{13}$$

and which provides a measure of the trade-off between the model's complexity (reflected in the number of estimated parameters, k) and the goodness-of-fit (reflected in the log-likelihood of the fit). The better estimation accuracy using the proposed model is quantitatively reflected in a smaller *AIC* value (121), when compared to 135 achieved by the linear model.

Based on obtained estimates, the in-field reliability is evaluated at different time moments, as depicted in Fig. 3. In order to address the reliability assessment uncertainty, we derived the confidence interval using the bootstrap method [13]. Simulation results reveal that not being able to capture accurately the degradation process nonlinearities, can result in an underestimation of the reliability, especially during the initial degradation period. This is turn may yield a less efficient lifetime management of the device whose reliability is being assessed. In the ICs case, for instance, being able to realistically asses the reliability status during the early in-field period, is of particular interest. This is because of the front-loaded nature (i.e., the highest extent of degradation is manifested during early operation, after which the degradation tends to saturate) of the front-end-of-life aging mechanisms, which imply that the lifetime management strategies are most effective during early life.

5. Case study

For expository purposes of proposed approach assessment of validity and potential applicability in reliability management frameworks, we provide subsequently a practical case study using the aging data of a PMOS transistor. To this end, we conducted accelerated aging simulation of a PMOS transistor implemented in PTM 45 nm technology [14]. As aging quantifier we employ the transistor threshold voltage, V_{th} [15]. The reliability analysis is carried in Cadence RelXpert and Virtuoso Spectre simulators [16], using the substrate and gate current, lifetime and AgeMos model parameters extracted in BSIMPro+ [17] for the PTM 45 nm technology. As concerns the environmental parameters, we used a temperature of 27 °C, and a power supply VDD = 1.0 V. We exposed the PMOS transistor to Negative Bias Temperature Instability (NBTI) and Hot Carrier Injection (HCI) wearout stress and adopted an EOL target (failure threshold *T*) of 9 years.

The percentage degradation of the transistor V_{th} is graphically illustrated in Fig. 4. The V_{th} time evolution, as obtained from Cadence simulation, serves as the real degradation data. Based on the V_{th} data, the noisy observations are then obtained in a similar manner with the synthetic example previously studied, specifically by sampling from the distribution $\mathcal{N}(V_{th}(t), 0.01)$. We derived the transistor EOL values, using the proposed approach and the linear model approach, at two different observation time moments: 1 year and 8 years, respectively. Based on the EOL values, the transistor remaining lifetime values were then obtained, each as the difference between the EOL time moment and the current observation time moment. The corresponding Probability Density Functions (PDFs) of the remaining useful lifetime values estimated with both proposed and linear approach, and the real remaining lifetime values obtained from Cadence, are depicted in Fig. 4 for comparison.

As it can be observed in Fig. 4, at the beginning of the transistor operating life, the uncertainty in the estimated remaining lifetime PDFs, under both proposed and the linear approach, is higher.

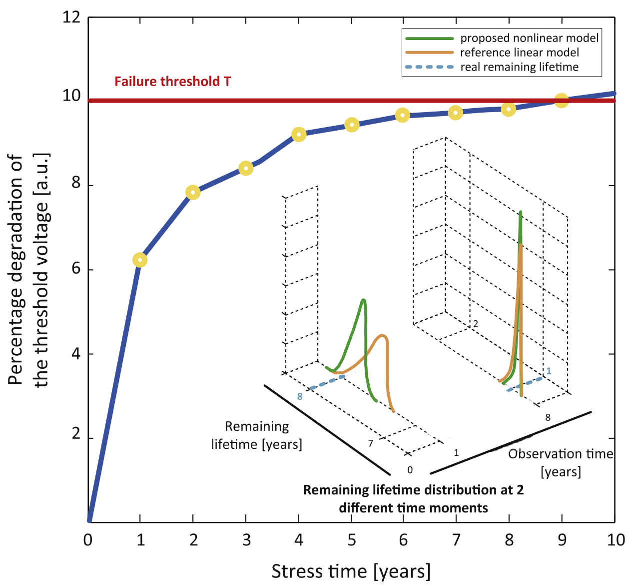

Fig. 4. The time evolution of the V_{th} degradation after 10-year simulation and the remaining lifetime distribution at two different observation time moments: 1 year and 8 years.

However, our model outperforms the linear counterpart, with a more precise estimation spread and a PDF mean value closer to the real transistor remaining lifetime value. The early EOL and implicitly the remaining lifetime estimation accuracy differences between the two approaches, can be attributed to the ability to capture the nonlinearities exhibited by the V_{th} degradation observations. As the circuit ages and more degradation observations become available, the EOL prediction uncertainty cones get narrower, and the differences between the two distributions become smaller.

When limited degradation observations are available, the accuracy of early EOL predictions is more sensitive to the selection of the prior distribution of $\theta = (\alpha, \sigma)$, which characterizes the degradation process, i.e., an inappropriate selection of these initial parameters, causes the predictions to be less accurate with smaller confidence intervals. Such is the case in the considered simulation setup which yields less accurate EOL predictions both for our approach and for the reference linear one, during the transistor early life, as illustrated by the two PDFs in Fig. 4 at 1 year observation time. However, the proposed approach takes into account the nonlinearities of the degradation process and is less sensitive to the selection of the prior distribution, exhibiting better adapting ability as far as the θ updating is concerned and, as a consequence, better prediction accuracy when compared to the linear model. Improved accuracy of EOL predictions during the early life stages, can be achieved if the prior distribution of $\theta = (\alpha, \sigma)$ parameters is restricted to meaningful values. However, for the current technology nodes with the afferent highly dynamic variability threats, precise knowledge based on experience with the same failure mechanisms in similar components may be harder to obtain.

As the amount of available degradation observations increases, the predictive ability improves for both approaches, as the posterior PDF becomes dominated by the likelihood given by (12), situation exemplified in Fig. 4 by the two PDFs at 8 years observation time.

The previously studied practical case, illustrates the significance of incorporating nonlinearity in the degradation process model when the underlying process is nonlinear, especially when EOL predictions are desired during the beginning of the device life, characterized by limited degradation history.

6. Conclusions

In this paper we proposed a Bayesian EOL and reliability estimation framework that takes into account the degradation process nonlinearity and uncertainty, from noisy observations. Based on the degradation history and the current measurement data, the degradation process parameters are updated via the Bayesian method. As such, future degradation evolution can be derived and the afferent reliability statistics estimated. Simulation results revealed that when compared to degradation processes with linear mean, the degradation process dynamics and the reliability evolution can be captured more accurately using the proposed framework.

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