# A Reverse Converter for the New 4-Moduli Set $\{2n+3, 2n+2, 2n+1, 2n\}$

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***Abstract*—In this paper, we propose a new 4-moduli set $\{2n+3, 2n+2, 2n+1, 2n\}$ that increases the dynamic range and the processing parallelism enabling efficient reverse conversion. First, we assume a general 4-moduli set $\{m_i\}_{i=1,4}$, $m_1 > m_2 > m_3 > m_4$, with the dynamic range $M = \prod_{i=1}^{4} m_i$ and introduce a modified Chinese Remainder Theorem (CRT) that requires mod-$m_4$ instead of mod-$M$ calculations. Subsequently, we further simplify the conversion process by focussing on the $\{2n+3, 2n+2, 2n+1, 2n\}$ moduli set, which has a common factor of 2. Given that for such a moduli set, CRT cannot be directly applied, we introduce a CRT based approach for this case, which first requires the conversion of $\{2n+3, 2n+2, 2n+1, 2n\}$ set into the moduli set with relatively prime moduli, i.e., $\left\{m_1, \frac{m_2}{2}, m_3, m_4\right\}$, valid for $n$ even, which are not multiples of 3. We demonstrate that such a conversion can be easily done and doesn't require the computation of any multiplicative inverses. For this case, the proposed CRT utilizes the same or slightly larger area when compared to other existing techniques but all the operations are mod-$m_4$. This outperforms state of the art CRTs in terms of the magnitude of the numbers involved in the calculation and due to this fact, our proposal results in less complex adders and multipliers.**

***Index Terms*—Residue Number System, 4-Moduli Set with Common factor, RNS-Decimal Converter, Chinese Remainder Theorem.**

## I. Introduction

Residue Number Systems (RNS) offer great potential for high-speed computer arithmetic due to their inherent properties such as parallelism, modularity, fault tolerance, and carry-free operations. These properties make them highly useful in Digital Signal Processing (DSP) applications where repeated additions and multiplications are required [1], [2], [11]. The major obstacle to the utilization of RNS is the overhead incurred in the conversions into and out of RNS. Relatively speaking, the forward conversion is a simpler and straightforward task whereas the reverse conversion involves considerable degree of complexity [11], [12]. Several converters have been proposed in the past [3], [4], [6]-[13] based on either the Chinese Remainder Theorem (CRT) or Mixed Radix Conversion (MRC). Extensively, different forms of three moduli sets have been studied with $\{2^n+1, 2^n, 2^n-1\}$ being the most popular one [3], [4], [5], [9].

The special moduli of the form $\{2n+2, 2n+1, 2n\}$ has been studied in [4], [5], [14]. This set is an extension of well studied $\{2^n+1, 2^n, 2^n-1\}$ set. When compared with $\{2^n+1, 2^n, 2^n-1\}$, the set $\{2n+2, 2n+1, 2n\}$ is particularly useful for decimal numbers which fall beyond the range specified by the $\{2^n+1, 2^n, 2^n-1\}$ set, resulting in the use of the next higher index for $n$ [4]. Another attractive property of the $\{2n+2, 2n+1, 2n\}$ set is that the numbers are consecutive enabling equal width multipliers to be used in the hardware implementation [5]. However, the dynamic range provided by the three moduli sets are insufficient in supporting high performance DSP applications requiring a large dynamic range and increased parallelism [11]. In this line of reasoning, we extend the moduli set $\{2n+2, 2n+1, 2n\}$ by adding $2n+3$ in order to obtain the 4-moduli superset $\{2n+3, 2n+2, 2n+1, 2n\}$. The new moduli set $\{2n+3, 2n+2, 2n+1, 2n\}$ can be seen like an extension of the 4-moduli sets $\{2^n-1, 2^n, 2^n+1, 2^{n+1}+1\}$ and $\{2^n-3, 2^n+1, 2^n-1, 2^n+3\}$ proposed in [11] and [13], respectively. For decimal numbers which fall beyond the range specified by these two 4-moduli sets, the moduli set $\{2n+3, 2n+2, 2n+1, 2n\}$ is of interest resulting in the utilization of the next higher index for $n$. Another attractive feature of the moduli set $\{2n+3, 2n+2, 2n+1, 2n\}$ is that two of the numbers share a common factor and also the numbers are consecutive enabling equal width multipliers to be used in the hardware implementation.

In this paper, we propose a new 4-moduli set $\{2n+3, 2n+2, 2n+1, 2n\}$ that increases the dynamic range and the processing parallelism enabling efficient reverse conversion. First, we assume a general 4-moduli set $\{m_i\}_{i=1,4}$, $m_1 > m_2 > m_3 > m_4$, with the dynamic range $M = \prod_{i=1}^{4} m_i$ and introduce a modified CRT that requires mod-$m_4$ instead of mod-$M$ calculations. Subsequently, we further simplify the conversion process by focussing on the new four superset $\{2n+3, 2n+2, 2n+1, 2n\}$, which has a common factor of 2. Given that for such a moduli set, CRT cannot be directly applied, we introduce a CRT based approach for this case, which first requires the conversion of $\{2n+3, 2n+2, 2n+1, 2n\}$ set into the moduli set with relatively prime moduli, i.e., $\left\{m_1, \frac{m_2}{2}, m_3, m_4\right\}$, valid for $n$ even which are not multiples of 3. We demonstrate that such a conversion can be easily done and doesn't require the computation of any multiplicative inverses. For this case, the proposed CRT utilizes the same or slightly larger area when compared to other existing techniques but all the operations are mod-$m_4$. This outperforms state of the art CRTs in terms of the magnitude of the numbers involved in the calculation and due to this fact, our proposal results in less complex adders and multipliers.

The rest of the article is organized as follows. In Section II, we introduce the necessary background. Section III presents the proposed algorithm. In Section IV, we evaluate the performance of the proposed scheme while the paper is concluded in Section V.

## II. Background

RNS is defined in terms of a set of relatively prime moduli $\{m_i\}_{i=1,n}$ such that gcd $(m_i, m_j) = 1$ for $i \neq j$, where gcd means the greatest common divisor of $m_i$ and $m_j$, while $M = \prod_{i=1}^{n} m_i$, is the dynamic range. The residues of a decimal number $X$ can be obtained as $x_i = |X|_{m_i}$ thus, it can be represented in RNS as $X = (x_1, x_2, x_3, ..., x_n)$, $0 \leq x_i < m_i$ . This representation is unique for any integer $X \in [0, M-1]$. We note here that in this paper, we use $|X|_{m_i}$ to denote the $X$ mod $m_i$ operation and the operator $\Theta$ to represent the operation of addition, subtraction, or multiplication. Given any two integer numbers $K$ and $L$ RNS represented by $K = (k_1, k_2, k_3, ..., k_n)$ and $L = (l_1, l_2, l_3, ..., l_n)$, respectively, $W = K \Theta L$, can be calculated as $W = (w_1, w_2, w_3, ..., w_n)$, where $w_i = |k_i \Theta l_i|_{m_i}$, for $i = 1, n$. This means that the complexity of the calculation of the $\Theta$ operation is determined by the number of bits required to represent the residues and not by the one required to represent the input operands.

For a moduli set $\{m_i\}_{i=1,n}$ with the dynamic range $M = \prod_{i=1}^{n} m_i$, the residue number $(x_1, x_2, x_3, ..., x_n)$ can be converted into the decimal number X, according to the CRT, as follows [1]:

$$X = \left| \sum_{i=1}^{n} M_i \left| M_i^{-1} x_i \right|_{m_i} \right|_M , \tag{1}$$

where $M = \prod_{i=1}^{n} m_i$, $M_i = \frac{M}{m_i}$, and $M_i^{-1}$is the multiplicative inverse of $M_i$ with respect to $m_i$. The New CRT [7] is formulated as follows:

$$X = x_1 + m_1 \left| w_1 x_1 + \sum_{i=2}^{n} w_i \left| M_i^{-1} x_i \right|_{m_i} \right|_{m_2 \ldots m_n} , \tag{2}$$

where $n > 1$, $w_i = \frac{M_i}{m_1}$ and $M_i^{-1}$ is the multiplicative inverse of $M_i$ with respect to $m_i$ and $M = \prod_{i=1}^{n} m_i$.

We note here that the moduli set $\{m_i\}_{i=1,n}$ must be pairwise relatively prime for Equation (1) to be directly used. For the $\{2n+3, 2n+2, 2n+1, 2n\}$ moduli set $2n+2$ and $2n$ share a common factor. This implies that to utilize Equation (1) in the conversion process this moduli set must be first mapped to a set of relatively prime moduli. If a moduli set is not pairwise relatively prime, then not every residue set $(x_1, x_2, x_3, ..., x_n)$ corresponds to a number and this results into inconsistency. As given in [1], a set of residues is consistent if and only if $|x_i|_k = |x_j|_k$ where $k = gcd(m_i, m_j)$ for all $i$ and $j$. If this holds true the decimal equivalent of $(x_1, x_2, x_3, ..., x_n)$ for moduli set which are not pairwise relatively prime can be computed as follows [1]:

$$|X|_{M_L} = \left| \sum_{i=1}^{n} \alpha_i x_i \right|_{M_L} , \tag{3}$$

where $M_L$ is the Lowest Common Multiple (LCM) of $\{m_i\}_{i=1,n}$, the set of moduli sharing a common factor, X is the decimal equivalent of $\{x_i\}_{i=1,n}$, $\alpha_i$ is an integer such that $|\alpha_i|_{\frac{M_L}{\mu_i}} = 0$ and $|\alpha_i|_{\mu_i} = 1$, and $\{\mu_i\}_{i=1,n}$ is a set of integers such that $M_L = \prod_{i=1}^{n} \mu_i$ and $\mu_i$ divides $m_i$. It should be noted that $\alpha_i$ may not exist for some $i$.

## III. Proposed Algorithm

The main idea behind our approach is to simplify Equation (1) by eliminating the large modulo $M$ and by removing the cost of computing $M_i^{-1}$. In this section, we demonstrate that the first one is possible for any 4-moduli RNS, while the second one can be achieved only for 4-moduli sets, which are not pairwise relatively prime.

We first introduce a modified CRT for any moduli set of length four, which doesn't require mod-$M$ computations.

*Theorem 1:* For a moduli set $\{m_i\}_{i=1,4}$, $m_1 > m_2 > m_3 > m_4$, the decimal equivalent X of the residues $(x_1, x_2, x_3, x_4)$ can be computed by using mod-$m_4$ (the smallest modulus) instead of the large mod-$M$ operations as:

$$X = (x_1 + x_2 + x_3) + m_1 m_2 m_3 | k_1 x_1 + k_2 x_2 + k_3 x_3 + \left| M_4^{-1} \right|_{m_4} x_4 |_{m_4}, \tag{4}$$

where $M_4^{-1}$ is the multiplicative inverse of $M_4$, $k_1 = \frac{\left(M_1 \left| M_1^{-1} \right|_{m_1} - 1\right)}{m_1 m_2 m_3}$, $k_2 = \frac{\left(M_2 \left| M_2^{-1} \right|_{m_2} - 1\right)}{m_1 m_2 m_3}$ and $k_3 = \frac{\left(M_3 \left| M_3^{-1} \right|_{m_3} - 1\right)}{m_1 m_2 m_3}$.

*Proof:* We utilize the lemmas presented in [7]:

Lemma 1: $|a m_1|_{m_1 m_2} = m_1 |a|_{m_2}$

Lemma 2: $M_1 \left| M_1^{-1} \right|_{m_1} = 1 + k_1 m_1 m_2 m_3$

Lemma 3: $M_2 \left| M_2^{-1} \right|_{m_2} = 1 + k_2 m_1 m_2 m_3$

Lemma 4: $M_3 \left| M_3^{-1} \right|_{m_3} = 1 + k_3 m_1 m_2 m_3$

Expanding Equation (1) for $n = 4$ we obtain:

$$X = |M_1 \left| M_1^{-1} \right|_{m_1} x_1 + M_2 \left| M_2^{-1} \right|_{m_2} x_2 + M_3 \left| M_3^{-1} \right|_{m_3} x_3 + M_4 \left| M_4^{-1} \right|_{m_4} x_4 |_{m_1 m_2 m_3 m_4} \tag{5}$$

Using Lemma 2 and 3 in the above equation, we obtain:

$$X = |(1 + k_1 m_1 m_2 m_3) x_1 + (1 + k_2 m_1 m_2 m_3) x_2 + (1 + k_3 m_1 m_2 m_3) x_3 + M_4 \left| M_4^{-1} \right|_{m_4} x_4 |_{m_1 m_2 m_3 m_4} \tag{6}$$

Further simplification gives:

$$X = (x_1 + x_2 + x_3) + |k_1 m_1 m_2 m_3 x_1 + k_2 m_1 m_2 m_3 x_2 + k_3 m_1 m_2 m_3 x_3 + M_4 \left| M_4^{-1} \right|_{m_4} x_4 |_{m_1 m_2 m_3 m_4} \tag{7}$$

Applying Lemma 1, we get:

$$X = (x_1 + x_2 + x_3) + m_1 m_2 m_3 | k_1 x_1 + k_2 x_2 + k_3 x_3 + M_4^* \left| M_4^{-1} \right|_{m_4} x_4 |_{m_4} \tag{8}$$

Here, $M_4^* = \frac{M_4}{m_1 m_2 m_3} = 1$, the equation then reduces to:

$$X = (x_1 + x_2 + x_3) + m_1 m_2 m_3 | k_1 x_1 + k_2 x_2$$

$$+k_3x_3 + \left|M_4^{-1}\right|_{m_4} x_4|_{m_4} \quad (9)$$

■

It can be observed that Equation (9) makes use of mod-$m_4$ (the smallest modulus) instead of mod-$M$ operations thus the magnitude of the involved values is smaller than in both the traditional CRT and the new CRT [7], given that $k_1$, $k_2$ and $k_3$ can be precomputed.

The next simplification step is the elimination of the $M_i^{-1}$. To achieve that, we restrict to the new superset $\{2n+3, 2n+2, 2n+1, 2n\}$. Let this set be represented by $\{m_1, m_2, m_3, m_4\}$ where $m_2$ and $m_4$ have a common factor of 2. Suppose that the set $\{m_1, m_2, m_3, m_4\}$ is mapped into a set of pairwise relatively prime moduli set $\{\mu_1, \mu_2, \mu_3, \mu_4\}$, the new dynamic range will be given by:

$$M_L = \prod_{i=1}^{4} \mu_i, \quad (10)$$

where $\mu_i$ is any chosen set of integers within the given moduli. It should be noted that $\mu_i$ may have several sets. As discuss earlier, for moduli set with a common factor, not every residue set corresponds to a number. Particularly, with the moduli set $\{2n+3, n+1, 2n+1, 2n\}$, even numbers $n$ that are multiple of 3 result into inconsistency. Thus, we choose a set of $\mu_i$ that is pairwise relatively prime for any even integer $n$ that is not a multiple of 3. The valid set is represented by $\{2n+3, n+1, 2n+1, 2n\}$. Next, we show that the computation of multiplicative inverses for this set can be eliminated for any even integer $n$, which is not a multiple of 3 using the following theorem:

*Theorem 2:* Given the moduli set $\{2n+3, 2n+2, 2n+1, 2n\}$ with $m_1 = 2n+3, m_2 = 2n+2, m_3 = 2n+1, m_4 = 2n$, the following hold true:

$$\left|M_2^{-1}\right|_{m_2} = \frac{n}{2} + 1, \quad (11)$$

$$\left|M_3^{-1}\right|_{m_3} = 2n. \quad (12)$$

*Proof:*

If it can be demonstrated that $\left|(\frac{n}{2}+1) \times (m_1m_3m_4)\right|_{m_2} = 1$, then $\frac{n}{2}+1$ is the multiplicative inverse of $m_1m_3m_4$ with respect to $m_2$. $|(\frac{n}{2}+1) \times (m_1m_3m_4)|_{m_2}$ is given by: $|(2n+3)(2n+1)(2n)(\frac{n}{2}+1)|_{n+1} = |(n+1)(4n^3+12n^2+5n)+2n^2+n|_{n+1} = ||(n+1)(4n^3+12n^2+5n)|_{n+1}+|2n^2+n|_{n+1}|_{n+1} = |0+1|_{n+1} = 1$, thus Equation (11) holds true.

In the same way if $|(2n) \times (m_1m_2m_4)|_{m_3} = 1$, then $2n$ is the multiplicative inverse of $(m_1m_2m_4)$ with respect to $m_3$. $|(2n) \times (m_1m_2m_4)|_{m_3} = 1$ is given by: $|(2n+3)(n+1)(2n)(2n)|_{2n+1} = |(2n+3)(n+1)(4n^2)|_{2n+1} = |(4n^3+8n^2)(2n+1)+(4n^2)|_{2n+1} = ||(4n^3+8n^2)(2n+1)|_{2n+1} + |(4n^2)|_{2n+1}|_{2n+1} = |0+1|_{2n+1} = 1$, thus Equation (12) holds true.

■

Next, it can be shown that the multiplicative inverses of $M_1$ and $M_4$ also exist and is demonstrated by the following theorems:

*Theorem 3:* For even numbers of the form $\{2, 8, 14, 20, 26, 32, ...\}$, represented by $n = \{6k-4\}_{k=1,2,3,...}$

$$\left|M_1^{-1}\right|_{m_1} = 4k-2, \quad (13)$$

$$\left|M_4^{-1}\right|_{m_4} = 2k-1, \quad (14)$$

*Proof:* From the Theorem, $|M_1^{-1}|_{m_1} = 4k-2 = 2(2k-1)$. Thus, $|M_1^{-1}|_{m_1} = 2|M_4^{-1}|_{m_4}$, then we shall show the proof of $|M_1^{-1}|_{m_1}$ only. $M_1 = \frac{m_2}{2}m_3m_4$, meaning that $M_1 = (n+1)(2n+1)(2n)$, for different values of $n$, $|M_1M_1^{-1}|_{m_1}$ will be given by: $n = 2$, when $|M_1^{-1}|_{m_1} = 2$, and also $|2(n+1)(2n+1)(2n)|_{2n+3} = |4n(2n^2+3n+1)|_{2n+3} = |4n^2(2n+3)+4n|_{2n+3} = ||4n^2(2n+3)|_{2n+3}+|4n|_{2n+3}|_{2n+3} = |0+1|_{2n+3} = 1$.

Similarly, when $n = 8$, $|M_1^{-1}|_{m_1} = 6$, and $|6(n+1)(2n+1)(2n)|_{2n+3} = ||12n^2(2n+3)|_{2n+3}+|12n|_{2n+3}|_{2n+3} = |0+1|_{2n+3} = 1$.

Again, when $n = 14$, $|M_1^{-1}|_{m_1} = 10$, and $|10(n+1)(2n+1)(2n)|_{2n+3} = ||20n^2(2n+3)|_{2n+3}+|20n|_{2n+3}|_{2n+3} = |0+1|_{2n+3} = 1$. Hence, if it is true for $n = 2, 8, 14$, then it will be true for any integer $n$ in this category. ■

*Theorem 4:* For even numbers of the form $\{4, 10, 16, 22, 28, 34, ...\}$, represented by $n = \{6k-2\}_{k=1,2,3,...}$

$$\left|M_1^{-1}\right|_{m_1} = 8k-2, \quad (15)$$

$$\left|M_4^{-1}\right|_{m_4} = 10k-3, \quad (16)$$

*Proof:* It can be proved in a similar manner to Theorem 3. ■

It should be noted that for any $n$-even that are not multiples of 3, the following expressions can be deduced from Theorem 2:

$$\left|M_2^{-1}\right|_{m_2} = \frac{m_2+2}{4}, \left|M_3^{-1}\right|_{m_3} = m_4. \quad (17)$$

Using Equation (17) and by proper substitutions in Theorem 1, we can particularize it for 4-moduli RNS sharing a common factor as follows:

*Corollary 1:* For the moduli set $\{2n+3, 2n+2, 2n+1, 2n\}$, the decimal equivalent $X$ of the residues $(x_1, x_2, x_3, x_4)$ can be computed as follows:

1) (**Using Theorem 3**):
$$X = (x_1+x_2+x_3) + m_1m_2m_3|k_1x_1 + k_2x_2 + k_3x_3 + (2k-1)x_4|_{m_4}, \quad (18)$$
where $k_1 = \frac{(m_2m_3m_4(4k-2)-1)}{m_1m_2m_3}$, $k_2 = \frac{(m_1m_3m_4(\frac{m_2+2}{4})-1)}{m_1m_2m_3}$, and $k_3 = \frac{(m_1m_2m_4(m_4)-1)}{m_1m_2m_3}$
2) ( **Using Theorem 4**):
$$X = (x_1+x_2+x_3) + m_1m_2m_3|k_1x_1 + k_2x_2 + k_3x_3 + (10k-3)x_4|_{m_4}, \quad (19)$$
where $k_1 = \frac{(m_2m_3m_4(8k-1)-1)}{m_1m_2m_3}$, $k_2 = \frac{(m_1m_3m_4(\frac{m_2+2}{4})-1)}{m_1m_2m_3}$ and $k_3 = \frac{(m_1m_2m_4(m_4)-1)}{m_1m_2m_3}$

| Metrics | CRT [1] | New CRT [7] | Our proposal |
|---|---|---|---|
| Area | 1 adder<br>4 multipliers | 1 adder<br>5 multipliers | 1 adder<br>5 multipliers |
| Dely | 1 addition<br>1 multiplication | 1 addition<br>2 multiplications | 1 addition<br>2 multiplications |
| Mod Optns | $m_1m_2m_3m_4$ | $m_2m_3m_4$ | $m_4$ |

Table I
PERFORMANCE COMPARISON

*Proof:* Trivial with proper substitutions for the values of $\left|M_2^{-1}\right|_{m_2}$ and $\left|M_3^{-1}\right|_{m_3}$ together with $\left|M_1^{-1}\right|_{m_1}$ and $\left|M_4^{-1}\right|_{m_4}$, which are obtained from Theorems 3 and 4. ■

## IV. PERFORMANCE EVALUATION

Clearly, it can be seen that the numbers involved in the multiplication are very small when compared to the numbers involved in both the direct traditional CRT and the New CRT [7] implementations. Additionally, the large modulo $M$ calculations are replaced by modulo calculations with the smallest modulus in the moduli set under consideration.

We take note here that in Table I, mod Optns stands for Modulo Operations. As indicated in Table I, In terms of area, our proposal requires the same area with the New CRT [7] whereas the traditional CRT [1] utilizes lesser area when compared to both the New CRT [7] and our approach. On the other hand, in terms of critical path delay, the CRT [1] requires 1 multiplication lesser than both the New CRT [7] and our technique but more important for the hardware complexity, the operands magnitude is significantly reduced by our proposal. More specifically, the modulo operation has been reduced from modulo $M = m_1m_2m_3m_4$ in [1] or $M = m_2m_3m_4$ in [7] to modulo $M = m_4$ in our scheme. This implies that our technique is manipulating smaller numbers when compared to the other techniques. The smaller the involved numbers in the calculation, the faster the arithmetic operations. Thus, our proposal is faster than the other techniques.

Finally, when compared to the existing similar 3-moduli set [4], [5], [14], the newly introduced four moduli set offers a larger dynamic range and a higher parallelism, which makes it more attractive for high performance computing.

## V. CONCLUSIONS

In this paper, we proposed a new 4-moduli set $\{2n+3, 2n+2, 2n+1, 2n\}$ that increases the dynamic range and the processing parallelism enabling efficient reverse conversion. First we assume a general 4-moduli set $\{m_i\}_{i=1,4}$, $m_1 > m_2 > m_3 > m_4$, with the dynamic range $M = \prod_{i=1}^{4} m_i$ and introduced a modified CRT that requires mod-$m_4$ instead of mod-$M$ calculations. This scheme can be utilized in conjunction with other well established $4-$moduli sets, e.g, $\{2^n - 1, 2^n, 2^n + 1, 2^{n+1} - 1\}$, $\{2^n - 1, 2^n, 2^n + 1, 2^{n+1} + 1\}$, $\{2^n - 3, 2^n + 1, 2^n - 1, 2^n + 3\}$ proposed in [10], [11], [12], respectively, and makes the CRT based conversion more effective as it reduces the magnitude of the values involved in the conversion thus the associated costs in area and delay. Subsequently, we further simplified the conversion process by focussing on $\{2n+3, 2n+2, 2n+1, 2n\}$ moduli set, which has a common factor of 2. Given that for such a moduli set, CRT cannot be directly applied, we introduced in a formal way a CRT based approach for this case, which requires the conversion of $\{2n+3, 2n+2, 2n+1, 2n\}$ set into moduli set with relatively prime moduli, i.e., $\left\{m_1, \frac{m_2}{2}, m_3, m_4\right\}$, when $n$ is even, which are not multiple of 3. We demonstrated that the moduli set transformation can be easily done and doesn't require the computation of any multiplicative inverses. For this case, the proposed CRT requires the same or slightly larger area when compared to other existing techniques but all the operations are mod-$m_4$. This outperforms state of the art CRTs in terms of the magnitude of the numbers involved in the calculation and due to this fact, our proposal results in less complex adders and multipliers. Finally, when compared to the existing similar 3-moduli set [4], [5], [14], this newly introduced four moduli set offers a larger dynamic range and a higher parallelism, which makes it more attractive for high performance computing.

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