A Residue to Binary Converter for the $\{2n+2, 2n+1, 2n\}$ Moduli Set

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Abstract*—In this paper, we investigate Residue Number System (RNS) to decimal conversion for a three moduli set with a common factor. We propose a new RNS to binary converter for the moduli set $\{2n+2, 2n+1, 2n\}$ for any even integer $n > 0$. First, we demonstrate that for such a moduli set, the computation of the multiplicative inverses can be eliminated. Secondly, we simplify the Chinese Remainder Theorem (CRT) to obtain a reverse conveter that uses mod-n instead of mod-$(2n+2)(2n)$ or mod-$2n$ required by other state of the art equivalent converters. Next, we present a low complexity implementation that does not require explicit use of the modulo operation in the conversion process as it is normally the case in the traditional CRT and other state of the art equivalent converters. In terms of area, our proposal requires four 2:1 adders and 2 multipliers while the best state of the art equivalent converter requires one 3:1 adder, two 2:1 adders, and four multipliers. In terms of critical path delay, our scheme requires 3 additions and 1 multiplication with mod-n operations whereas the best state of the art equivalent converter requires 2 additions and 2 multiplications with mod-$2n$ operations. Consequently, our scheme outperforms state of the art converters in terms of area and delay. Moreover, due to the fact that our scheme operates on smaller magnitude operands, it requires less complex adders and multipliers, which potentially results in even faster and smaller implementations.

***Index Terms*—Residue Number System, RNS-Decimal Conversion, Moduli Set With Common Factors, Multiplicative Inverses, Chinese Remainder Theorem.**

I. Introduction

The Residue Number System (RNS) has interesting inherent features such as parallelism, modularity, fault tolerance, and carry free operations. These features make RNS to be widely used in Digital Signal Processing (DSP) applications such as digital filtering, convolution, fast Fourier transform, and image processing [4], [10]. For successful application of RNS, data conversion must be very fast so that the conversion overhead doesn't nullify the RNS advantages. Data Conversion, which is usually based on either the Chinese Remainder Theorem (CRT) [1], [3], [4], [6], [7], [9] or the Mixed Radix Conversion (MRC) [2], [12] has been actively investigated. The RNS for a three moduli set has been studied for a long time with $\{2^n+1, 2^n, 2^n-1\}$ being the most popular one [7]. However, the moduli set $\{2n+2, 2n+1, 2n\}$ is a strong alternative candidate for decimal numbers which fall beyond the range specified by the $\{2^n+1, 2^n, 2^n-1\}$ moduli set resulting in the use of next higher index for n [7], [8], and [9]. The moduli set $\{2n+2, 2n+1, 2n\}$ is desirable because the numbers are consecutive, enabling nearly equal width adders and multipliers in the hardware implementation and also two of the numbers share a common factor. Based on the weight concepts, the decoding of RNS numbers for the moduli set $\{2n+2, 2n+1, 2n\}$ has been presented in [9].

In this paper, we propose a new RNS to binary converter for the moduli set $\{2n+2, 2n+1, 2n\}$ for any even integer $n > 0$. First, we demonstrate that for such a moduli set, the computation of the multiplicative inverses can be eliminated. Secondly, we simplify the Chinese Remainder Theorem (CRT) to obtain a reverse conveter that uses mod-n instead of mod-$(2n+2)(2n)$ or mod-$2n$ required by other state of the art equivalent converters. Next, we present a low complexity implementation that does not require the explicit calculation of modulo operation in the conversion process as it is normally the case in the traditional CRT and other state of the art equivalent converters. In terms of area, our proposal requires four 2:1 adders and 2 multipliers while the best state of the art equivalent converter [4] requires one 3:1 adder, two 2:1 adders, and four multipliers. In terms of critical path delay, our scheme requires 3 additions and 1 multiplication with mod-n operations whereas the converter in [4] requires 2 additions and 2 multiplications with mod-$2n$ operations. Moreover, due to the fact that our scheme operates on smaller magnitude operands, it requires less complex adders and multipliers, which potentially results in even faster and smaller implementations.

The rest of the article is organised as follows: Section II presents the necessary background. In Section III we describe the proposed algorithm. Section IV presents the hardware realization of the proposed algorithm and a comparison with the state of the art, while the paper is concluded in Section V.

II. Background

RNS is defined in terms of a set of relatively prime moduli set $\{m_i\}_{i=1,n}$ such that $\gcd(m_i, m_j) = 1$ for $i \neq j$, where gcd means the greatest common divisor of m_i and m_j, while $M = \prod_{i=1}^{n} m_i$, is the dynamic range. The residues of a decimal number X can be obtained as $x_i = |X|_{m_i}$ thus X can be represented in RNS as $X = (x_1, x_2, x_3 ..., x_n)$, $0 \leq x_i < m_i$. This representation is unique for any integer $X \in [0, M-1]$. We note here that in this paper we use $|X|_{m_i}$ to denote the X mod m_i operation and the operator Θ to represent the operation of addition, subtraction, or multiplication. Given any two integer numbers K and L RNS represented by $K = (k_1, k_2, k_3, ..., k_n)$ and $L = (l_1, l_2, l_3, ..., l_n)$, respectively,

$W = K\Theta L$, can be calculated as $W = (w_1, w_2, w_3, ..., w_n)$, where $w_i = |k_i \Theta l_i|_{m_i}$, for $i = 1, n$. This means that the complexity of the calculation of the Θ operation is determined by the number of bits required to represent the residues and not by the one required to represent the input operands.

For a moduli set $\{m_i\}_{i=1,n}$ with the dynamic range $M = \prod_{i=1}^{n} m_i$, the residue number $(x_1, x_2, x_3, ..., x_n)$ can be converted into the decimal number X, according to the Chinese Reminder Theorem, as follows [10]:

$$X = \left| \sum_{i=1}^{n} M_i \left| M_i^{-1} x_i \right|_{m_i} \right|_M , \quad (1)$$

where $M = \prod_{i=1}^{n} m_i$, $M_i = \frac{M}{m_i}$, and M_i^{-1} is the multiplicative inverse of M_i with respect to m_i. We note here that the moduli set $\{m_i\}_{i=1,n}$ must be pairwise relatively prime for Equation (1) to be directly used. The moduli set $\{2n+2, 2n+1, 2n\}$ has a common factor of 2. This implies that to utilize Equation (1) in the conversion this moduli set must be first mapped to a set of relatively prime moduli. If a moduli set is not pairwise relatively prime, then not every residue set $(x_1, x_2, x_3, ..., x_n)$ corresponds to a number and this results into inconsistency. As discussed in [10], a set of residues is consistent if and only if $|x_i|_k = |x_j|_k$ where $k = gcd(m_i, m_j)$ for all i and j. If this holds true the decimal equivalent of $(x_1, x_2, x_3, ..., x_n)$ for moduli set which are not pairwise relatively prime can be computed as follows:

$$|X|_{M_L} = \left| \sum_{i=1}^{n} \alpha_i x_i \right|_{M_L} , \quad (2)$$

where M_L is the Lowest Common Multiple (LCM) of $\{m_i\}_{i=1,n}$, the set of moduli sharing a common factor, X is the decimal equivalent of $\{x_i\}_{i=1,n}$, α_i is an integer such that $|\alpha_i|_{\frac{M_L}{\mu_i}} = 0$ and $|\alpha_i|_{\mu_i} = 1$, and $\{\mu_i\}_{i=1,n}$ is a set of integers such that $M_L = \prod_{i=1}^{n} \mu_i$ and μ_i divides m_i. It should be noted that α_i may not exist for some i. In [4], Equation (2) has been represented as:

$$|X|_{M_L} = \left| \sum_{i=1}^{n} \beta_i \left| \beta_i^{-1} \right|_{\mu_i} x_i \right|_{M_L} , \quad (3)$$

where $M_L = LCM\{m_i\}_{i=1}^{n} = \prod_{i=1}^{n} \mu_i$, $\beta_i = \frac{M_L}{\mu_i}$, $\left|\beta_i^{-1}\right|_{\mu_i}$ is the multiplicative inverse of β_i with respect to μ_i.

For the moduli set under investigation, the following expressions have been derived as the decimal equivalent of the residues (x_1, x_2, x_3) in [9] and [7], respectively:

$$X = \left| \frac{m_2 m_3}{2} x_1 - m_1 m_3 x_2 + \frac{m_1 m_2}{2} x_3 \right|_{M_L} \quad (4)$$

$$X = x_2 + m_2 \left| (x_2 - x_3) + (x_1 - 2x_2 + x_3) \frac{m_1}{2} \right|_{\frac{m_1}{2} m_3} \quad (5)$$

For the same moduli set, a reverse converter that outperforms [7] was presented in [4] and represented by the following expression:

$$X = (x_1 + x_2) + \frac{m_1 m_2}{2} \left| k_1 x_1 + k_2 x_2 + \frac{m_1}{2} x_3 \right|_{m_3} , \quad (6)$$

where

$$k_1 = \frac{2\left((m_2 m_3)(\frac{m_3}{4} + 1) - 1\right)}{(m_1 m_2)},$$

$$k_2 = \frac{2\left(\frac{(m_1 m_3)}{2}(m_2 - 2) - 1\right)}{m_1 m_2}.$$

In the following section we present a reverse converter for the moduli set $\{2n+2, 2n+1, 2n\}$ by simplifying Equation (1). The resulting converter uses smaller modulo operation when compared to Equations (4), (5), and outperforms, in terms of both area and speed, the reverse converter presented in [4] represented by Equation (6).

III. Proposed Algorithm

Given the RNS number (x_1, x_2, x_3) with respect to the moduli set $\{m_1, m_2, m_3\}$ in the form $\{2n+2, 2n+1, 2n\}$, the proposed algorithm computes the decimal equivalent of this RNS number based on further simplification of the well-known traditional CRT. First, we show that the computation of the multiplicative inverses can be eliminated for this moduli set. Next, we obtain a reverse converter that uses modulo-$\frac{m_3}{2}$ instead of modulo-$\frac{m_1}{2}m_3$ or modulo-m_3 used by the state of the art equivalent converters. We then propose a low complexity implementation that does not require explicit use of the modulo operation at the final stage of computation. It should be noted that Equation (1) cannot be directly used for the conversion since in the moduli set $\{2n+2, 2n+1, 2n\}$, the moduli $2n+2$ and $2n$ share a common factor of 2. The moduli set must be first mapped into a set of relatively prime integers. In [9], it has been demonstrated that such a mapping can easily be done and that the set of relatively prime moduli for $\{2n+2, 2n+1, 2n\}$ moduli set, for any even integer $n > 0$, is given by $\{n+1, 2n+1, 2n\}$, meaning that the new moduli set is $\left\{\frac{m_1}{2}, m_2, m_3\right\}$.

Theorem 1: Given the moduli set $\{2n+2, 2n+1, 2n\}$ with $m_1 = 2n+2, m_2 = 2n+1, m_3 = 2n$, the following hold true:

$$\left|(\frac{m_1}{2} m_2)^{-1}\right|_{m_3} = n+1, \quad (7)$$

$$\left|(m_2 m_3)^{-1}\right|_{\frac{m_1}{2}} = \frac{n}{2} + 1, \quad (8)$$

$$\left|(\frac{m_1}{2} m_3)^{-1}\right|_{m_2} = 2n - 1. \quad (9)$$

Proof: If it can be demonstrated that $\left|(n+1) \times (\frac{m_1}{2} m_2)\right|_{m_3} = 1$, then $(n+1)$ is the multiplicative inverse of $(\frac{m_1}{2} m_2)$ with respect to m_3. $\left|(n+1) \times (\frac{m_1}{2} m_2)\right|_{m_3}$ is given by: $|(n+1)(n+1)(2n+1)|_{2n} = \left|(2n^3 + 5n^2 + 4n + 1)\right|_{2n} = \left|\left|2n(n^2 + \frac{5n}{2})\right|_{2n} + |2(2n)|_{2n} + |1|_{2n}|_{2n} = |0 + 0 + 1|_{2n} = 1$, thus Equation (7) holds true.

In the same way if $\left|(\frac{n}{2} + 1) \times (m_2 m_3)\right|_{\frac{m_1}{2}} = 1$, then $(\frac{n}{2} + 1)$ is the multiplicative inverse of $(m_2 m_3)$ with respect

to $\frac{m_1}{2}$. $\left|(\frac{n}{2}+1)\times(m_2m_3)\right|_{\frac{m_1}{2}}$ is given by: $|(\frac{n}{2}+1)(2n+1)(2n)|_{n+1} = |2n^3+5n^2+2n|_{n+1} = ||2n^2(n+1)|_{n+1} + |3n^2+2n|_{n+1}|_{n+1} = |0+1|_{n+1} = 1$, thus Equation(8) holds true.

Again, if $|(2n-1)\times(\frac{m_1}{2}m_3)|_{m_2} = 1$, then $2n-1$ is the multiplicative inverse of $(\frac{m_1}{2}m_3)$ with respect to m_2. $|(2n-1)\times(\frac{m_1}{2}m_3)|_{m_2}$ is given by:
$|(2n-1)(n+1)(2n)|_{2n+1} = |4n^3+2n^2-2n|_{2n+1} = ||2n^2(2n+1)|_{2n+1} + |-2n|_{2n+1}|_{2n+1} = |0+1|_{2n+1} = 1$, thus Equation (9) holds true. ■

As stated in Section II, for moduli sets with a common factor, not all remainder sets are valid numbers. The following proposition state the condition for a 3-residue set to represent a valid number.

Proposition 1: For RNS with the moduli set $\{m_1, m_2, m_3\}$ sharing a common factor, (x_1, x_2, x_3) represents a valid number if and only if $(x_1 + x_3)$ is even.

Proof: This proposition has been proved in [7]. ■

The following theorem introduces a simplified way to compute the decimal equivalent of the RNS number (x_1, x_2, x_3) with respect to the moduli set $\{m_1, m_2, m_3\}$ in the form $\{2n+2, 2n+1, 2n\}$ for any even integer $n > 0$.

Theorem 2: The decimal equivalent of the RNS number (x_1, x_2, x_3) with respect to the moduli set $\{m_1, m_2, m_3\}$ in the form $\{2n+2, 2n+1, 2n\}$ for any even integer $n > 0$ is computed as follows:

$$X = \left|\frac{m_2m_3}{2}x_1 - m_1m_3x_2 + \frac{m_1m_2}{2}x_3\right|_{M_L}, \quad (10)$$

where $M_L = \frac{m_1m_2m_3}{2}$.

Proof: For $n = 3$, Equation (1) becomes:

$$X = \left|\sum_{i=1}^{3} M_i \left|M_i^{-1}x_i\right|_{m_i}\right|_{M_L}. \quad (11)$$

By substituting Equations (7), (8), and (9) into Equation (11) we obtain the following:

$$\begin{aligned} X &= \left|(m_2m_3)\left(\frac{m_3}{4}+1\right)x_1 + \left(\frac{m_1}{2}m_3(m_2-2)\right)x_2 \right. \\ &\quad \left. + \left(\frac{m_1}{2}m_2\right)\frac{m_1}{2}x_3\right|_{M_L} \\ &= \left|\left(\frac{m_2m_3m_3}{4}\right)x_1 + m_2m_3x_1 + \frac{m_1m_2m_3}{2}x_2 \right. \\ &\quad \left. - m_1m_3x_2 + \frac{m_1m_1m_2}{4}x_3\right|_{M_L} \\ &= \left|\left(\frac{m_2m_3}{4}\right)x_1(m_1-2) + m_2m_3x_1 + M_Lx_2 \right. \\ &\quad \left. - m_1m_3x_2 + \frac{m_1m_2}{4}x_3(m_3+2)\right|_{M_L} \end{aligned}$$

Further simplifications give:

$$\begin{aligned} X &= \left|\left|\frac{M_L}{2}(x_1+x_3)\right|_{M_L} + \left|(\frac{m_2m_3}{2})x_1\right|_{M_L} \right. \\ &\quad \left. - |m_1m_3x_2|_{M_L} + \left|(\frac{m_1m_2}{2})x_3\right|_{M_L}\right|_{M_L} \quad (12) \end{aligned}$$

Since each of the terms $\frac{m_2m_3}{2}x_1$, $m_1m_3x_2$, and $\frac{m_1m_2}{2}x_3$ in Equation (12) is positive and less than M_L and also from Proposition I, $(x_1 + x_3)$ must always be even, which implies that, $\left|(x_1+x_3)\frac{M_L}{2}\right|_{M_L} = 0$. Equation (12) therefore reduces to:

$$X = \left|\frac{m_2m_3}{2}x_1 - m_1m_3x_2 + \frac{m_1m_2}{2}x_3\right|_{M_L} \quad (13)$$

Thus, Equation (10) holds true. ■

This equation is exactly the same as Equation (4) previously proved in [9] using the weight concepts. We propose to further simplify Equation (10) using the following theorem:

Theorem 3: The decimal equivalent of the RNS number (x_1, x_2, x_3) with respect to the moduli set $\{m_1, m_2, m_3\}$ in the form $\{2n+2, 2n+1, 2n\}$ for any even integer $n > 0$ is computed as follows:

$$\begin{aligned} X &= (x_2-x_1)m_1 + x_1 \\ &\quad + m_1m_2\left|\frac{(x_1+x_3)}{2} - x_2\right|_{\frac{m_3}{2}} \quad (14) \end{aligned}$$

Proof: To prove this theorem we use the following lemma presented in [11]:

$$|am_1|_{m_1m_2} = m_1\left|a\right|_{m_2}. \quad (15)$$

From Equation (10), we have

$$X = \left|\frac{m_2m_3}{2}x_1 - m_1m_3x_2 + \frac{m_1m_2}{2}x_3\right|_{M_L}$$

Putting $m_3 = m_2 - 1$ in the above equation, we obtain:

$$\begin{aligned} &= \left|\frac{m_2m_3}{2}x_1 - m_1x_2(m_2-1) + \frac{m_1m_2}{2}x_3\right|_{M_L} \\ &= m_1x_2 \\ &\quad + \left|\frac{m_2m_3}{2}x_1 - m_1m_2x_2 + \frac{m_1m_2}{2}x_3\right|_{\frac{m_1m_2m_3}{2}} \end{aligned}$$

Applying Equation (15) to the above equation gives:

$$\begin{aligned} X &= m_1x_2 \\ &\quad + m_2\left|\frac{m_3}{2}x_1 - m_1x_2 + \frac{m_1}{2}x_3\right|_{\frac{m_1m_3}{2}} \quad (16) \end{aligned}$$

Putting $m_3 = m_1 - 2$ in the above equation, we obtain:

$$\begin{aligned} &= m_1x_2 \\ &\quad + m_2\left|\frac{(m_1-2)}{2}x_1 - m_1x_2 + \frac{m_1}{2}x_3\right|_{\frac{m_1m_3}{2}} \\ &= m_1x_2 - m_2x_1 \\ &\quad + m_2\left|m_1\frac{(x_1+x_3)}{2} - m_1x_2\right|_{\frac{m_1m_3}{2}} \end{aligned}$$

Applying Equation (15) to the above equation gives:

$$\begin{aligned} X &= m_1x_2 - x_1(m_1-1) \\ &\quad + m_1m_2\left|\frac{(x_1+x_3)}{2} - x_2\right|_{\frac{m_3}{2}} \end{aligned}$$

Further simplifications give:

$$\begin{aligned} X &= (x_2-x_1)m_1 + x_1 \\ &\quad + m_1m_2\left|\frac{(x_1+x_3)}{2} - x_2\right|_{\frac{m_3}{2}} \end{aligned}$$

Thus, Equation (14) holds true. ■

It can be observed that Equation (14) is processing smaller numbers when compared to Equations (4), (5), and (3), thus the magnitude of the involved values in the proposed conversion is smaller than the one in state of the art equivalent converters.

IV. Hardware Realization

The hardware realization of the proposed scheme is depicted by Figure 1. The implementation follows Equation (14) but the following should be noted. At a first glance, D is a 3:1 adder. However, the extra input x_2 can be embedded into the partial product matrix of the m_1 multiplier according to the merged arithmetic principle. Furthermore, the modulo-$\frac{m_3}{2}$ operation associated with the adder C doesn't have to be explicitly computed. It can be replaced by at most one corrective addition.

In order to demonstrate that no explicit modulo operation is required by our proposal, we analyze the two possible extreme cases as follows:

Case 1: $(x_1 + x_3) = 0$ and $x_2 = 2n$. This results in the most negative value one may get. In this case Equation (14) reduces to $|-x_2|_{\frac{m_3}{2}}$. To perform the modulo $\frac{m_3}{2}$ operation, we need to do corrective additions. Given that $m_3 + (-x_2) = (2n) + (-2n) = 2n - 2n = 0$, for any positive even integer n, only one corrective addition with m_3 is required to compute the modulo.

Case 2: $(x_1 + x_3)$ is even and has the maximum possible value and x_2 is zero. This is the largest positive value one may get and Equation (14) reduces to $|\frac{(x_1+x_3)}{2}|_{\frac{m_3}{2}}$. Given that $m_3 - \frac{(x_1+x_3)}{2} = (2n) - \frac{(2n+1+2n-1)}{2} = 2n - 2n = 0$ the maximum sum in the modulo adder cannot exceed m_3, thus only one correction is required. This means that the modulo m_3 operation can be implemented with at most one corrective addition.

Figures 2 and 3 describe the hardware realization of the converters proposed in [4] and [7], respectively. The area, the delay, and the modulo operations required by our proposal and the one in [4] and [7] are summarized in Table I. As one can observe in the Table, our proposal requires less delay and operates on smaller magnitude operands with the same or less area. In particular, our proposal requires four 2:1 adders and two multipliers, Figure 2 requires one 3:1 adder, two 2:1 adders, and four multipliers while Figure 3 requires one 3:1 adder, three 2:1 adders and two multipliers. In terms of critical path delay, our scheme requires 3 additions and 1 multiplication with mod-n operations, the converter in [4] requires 2 additions and 2 multiplications with mod-$2n$ operations whereas the converter in [7] requires 3 additions, 2 multiplications with mod-$\frac{m_1}{2}m_3$ operations. Consequently, the new converter introduced in this paper requires less delay with the same or less area. Moreover, due to the fact that our scheme operates on smaller magnitude operands, it requires less complex adders and multipliers, which potentially results in even faster and smaller implementations.

Metrics	[7]	[4]	Our proposal
Area	4 adders 2 multipliers	3 adders 4 multipliers	4 adders 2 multipliers
Delay	3 additions 2 multiplications	2 additions 2 multiplications	3 additions 1 multiplication
Mod operations	$\frac{m_1}{2}m_3$	m_3	$\frac{m_3}{2}$

Table I
Performance Comparison

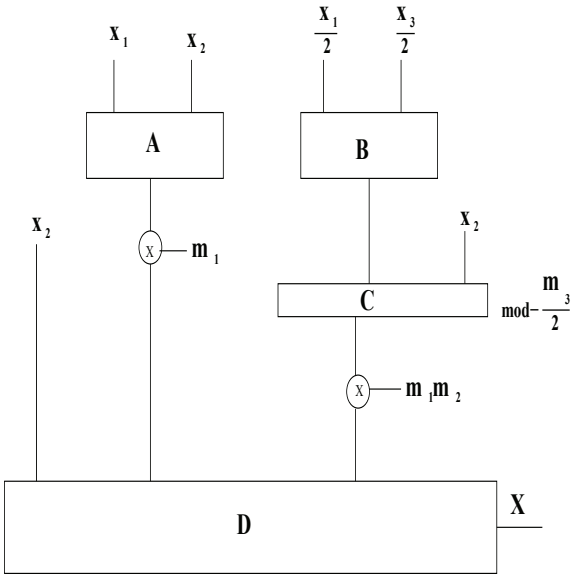

Figure 1. Hardware Realization of Our Proposal

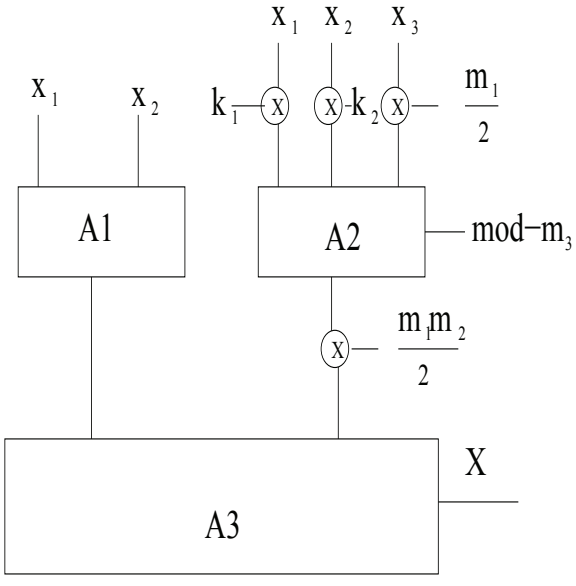

Figure 2. Converter Data Path for [4]

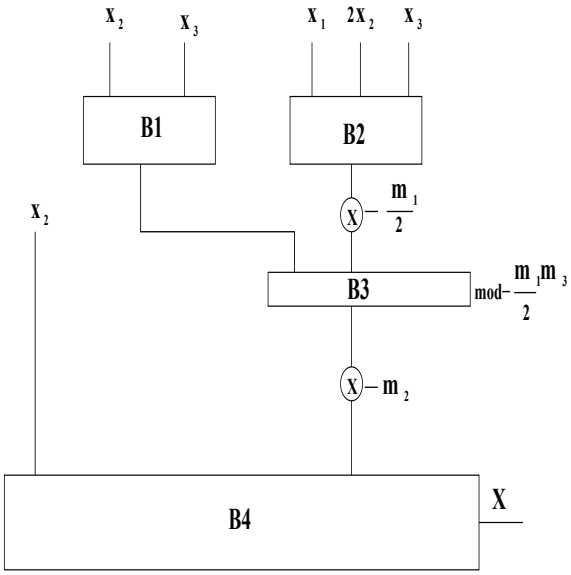

Figure 3. Converter Data Path for [7]

V. Conclusions

In this paper, we investigated RNS to decimal conversion which is an important issue concerning the utilization of RNS numbers in DSP applications. We proposed a new RNS to decimal converter for the moduli set $\{2n+2, 2n+1, 2n\}$ for any even integer $n > 0$. First, we demonstrated that for such a moduli set, the computation of multiplicative inverses can be eliminated. Secondly, we simplified the Chinese Remainder Theorem (CRT) to obtain a reverse conveter that uses mod-n instead of mod-$(2n+2)(2n)$ and mod-$2n$ required by other state of the art equivalent converters. Next, we presented a low complexity implementation that does not require the explicit use of the modulo operation in the conversion process as it is normally the case in the traditional CRT and other state of the art equivalent converters. In terms of area, our proposal requires four 2:1 adders and 2 multipliers while the best state of the art equivalent converter requires one 3:1 adder, two 2:1 adders, and four multipliers. In terms of critical path delay, our scheme requires 3 additions and 1 multiplication with mod-n operations whereas the best state of the art equivalent converter requires 2 additions and 2 multiplications with mod-$2n$ operations. Consequently, our scheme outperforms the best state of the art converter in terms of area and delay. Moreover, due to the fact that our scheme operates on smaller magnitude operands, it requires less complex adders and multipliers, which potentially results in even faster and smaller implementations.

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