# Reconfigurable Universal Adder 

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#### Abstract

In this paper we present a novel adder/subtracter arithmetic unit that combines Binary and Binary Code Decimal (BCD) operations. The proposed unit uses effective addition/subtraction operations on unsigned, sign-magnitude, and various complement representations. Our design overcomes the limitations of previously reported approaches that produce some of the results in complement representation when operating on sign-magnitude numbers. The proposal can be implemented in ASIC as a run time configurable unit as well as in reconfigurable technology in form of a run-time reconfigurable engine. When reconfigurable technology is considered, a preliminary estimation indicates that $40 \%$ of the hardware resources are shared by the different operations. This makes the proposed unit highly suitable for reconfigurable platforms with partial reconfiguration support. The proposed design together with some classical adder organizations were compared after synthesis targeting 4vfx60ff672-12 Xilinx Virtex 4 FPGA. Our design achieves a throughput of 82.6 MOPS with almost equivalent area-time product when compared to the other proposals.


## 1 Introduction

Various applications, e.g. commercial and financial electronic transactions [12], internet [19] and industrial control [22] require precise arithmetic for different data representation formats. Binary arithmetic is not capable of expressing exactly fractional numbers like, 0.2 , and therefore, Binary Decimal Arithmetic emerges as a possible solution [9] to avoid binary approximations [13, 3]. When performing decimal operations on traditional binary based hardware, excessive delays are introduced due to the software emulations, conversions and corrections, typically 100 to 1000 times slower [8]. Therefore, flexible hardware solutions for both decimal and binary processing are considered in this paper. We assume universal units similar to those
presented in [25] that are capable of performing various related operations sharing the same hardware. More specifically, the main contributions of this paper are:

- Twelve related operations were collapsed into a single Universal Adder (UA) hardware unit using UniversalNotation ${ }^{i}$. The proposed structure uses the effective addition/subtraction approach similar to [24] for both, binary and BCD notations.
- High hardware utilization: approximately $40 \%$ of the hardware resources implementing the 12 different operations are reused. This makes the unit a good candidate for implementation on partially reconfigurable platforms such as [26].

The remainder of this paper is organized as follows. Section 2 outlines the background and presents the related work. Section 3, exhibits with details the Reconfigurable Universal Adder design. Section 4 presents the experimental results and introduces the analysis in terms of used area and delay. Finally, Section 5 concludes the paper.

## 2 Background

Decimal digits are usually converted to binary representation as follows: the 8421 BCD code uses only the first ten combinations of a 4-bit binary code to represent each decimal digit. The remaining encodings ( 1010 to 1111) corresponding to decimal ( 10 to 15 ) are left unused when decimal computing is considered. Assuming addition of two decimal numbers $N 1$ and $N 2$ their $S U M$ can exceed $1001_{2}(S U M>9)$. When such a situation occurs, correcting addition using $0110_{2}$ (6) is required, so the complete operation becomes $S U M=N 1+N 2+0110_{2}$. Please note that in case of subtraction N 2 can be the complement representation of the original operand. In other words, decimal subtraction introduces additional processing. Recall

[^0]that a complement $(C N)$ of an $n$-bit number $N$ is computed by: $C N=r^{n}-N$; where $r$ is the radix of the number ( $r=2$ for binary, and $r=10$ for decimal). Thus, the ten's complement of an $x$-digit number $N$, is expressed by $C N_{N}=10^{x}-N$. When we consider only a single digit $D$ (as in the case with BCD), the computation is simplified to $C N_{D}=10-D$. Nevertheless, BCD encoding do not include a code for 10 and for this reason a nine's complement representation is used. In this case the computation becomes: $C N_{D}=10-D=9-D+1$.

Some helpful techniques used to compute a nine's complement of a digit are [21]: Pre-complement. In this approach the subtrahend is one-complemented, a $1010_{2}$ value is added for correction, and finally the generated carries are discarded. Post-complement. In this case, a binary $0110_{2}$ value is added to the subtrahend operand, the result is onecomplemented and the generated carries are used.

### 2.1 Related work

Many researchers addressed the problem of decimal arithmetic in the past. Early solutions proposed customized decimal adders, like Schmookler et al. [20] and Adiletta et al. [1]. Combined Binary and BCD adders were designed by Levine et al. [18] and Anderson [2], while true decimal sign-magnitude adder/substracter was presented by Grupe [14]. The latter used 3-binary-adders along with additional logic, achieving similar results as presented in this paper. An area efficient sign-magnitude adder was developed by Hwang [17]. In his approach two additional conversions are introduced before and after the binary addition. This is somehow similar to other proposals, e.g. [14]. The novelty in Hwang's proposal comes with the separation of the binary and the decimal results, using a multiplexor to select the correct output as is depicted in Figure 1.


Figure 1. Hwang's proposal [17]
Flora [11] presents an adder that process concurrently two different results, one assuming the presence of an input carry and the other assuming no carry in available. This is following the principle of carry select adders [4], an approach actually used in several state of the art units. To cope with the area overhead, another compact design employing a single adder was introduced by Fischer et al. [10]
depicted in Figure 2. This unit encodes ("edits") the incoming operands as well as the adder result to achieve the desired functionality.


Figure 2. Fischer's proposal [10]
During the last decade various binary and BCD adder/subtracter units for the IBM S/390, G4, G5 and G6 microprocessors [7] were developed. Following this line, the eServer z900 processor [23] includes a combined binary/decimal arithmetic unit working with binary-coded decimal numbers in sign-magnitude representation. The z900 unit was designed to operate in a single cycle, nevertheless a correction of the results is required in some of the cases [5]. For example, when an effective subtraction operation is performed, they need to calculate the complement value of the result, hence increasing the latency. The approach used to construct this decimal unit is based on the work presented in [15], which is shown in Figure 3. Recently, a similar approach was presented by Haller et al. [16]. In this work the carry chain is optimized resulting in a slight delay improvement with an increased area of the unit.


Figure 3. Haller's proposal (z900)

The following Table 1, summarizes the fundamental characteristics of the adders discussed above. The subset of the proposals aiming at signed arithmetic do not produce the correct result and need and additional complement operation in the cases when the subtrahend is greater than the minuend causing an additional cycle. As will be shown in the text to follow our approach does not have this disadvantage.

Table 1. Adders - data representation

| Adder | BCD |  | Binary |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Addition | Subtraction | Addition | Subtraction |
| Schmookler [20] | U | - | - | - |
| Addileta [1] | U | - | - | - |
| Levine [18] | U | - | U | - |
| Anderson [2] | U | - | U | - |
| Grupe [14] | S | S | S | S |
| Hwang [17] | S | ten's | S | two's |
| Flora [11] | S | ten's | S | two's |
| Fischer [10] | S | ten's | S | two's |
| z900 [5] | S | ten's | S | two's |
| Haller2006 [16] | S | ten's | S | two's |

U: unsigned; S: sign-magnitude. Two's and ten's: for complement representation. The last two represent the inconsistency to drive the correct result for $(N 2>N 1)$.

## 3 Reconfigurable Universal Adder

The main goal of our work was to create an adder capable of carrying out decimal operations using effective addition/subtraction scheme, overcoming the limitations of the adders presented in the related work section. To achieve this we used the $\mathrm{S} / 370$ sign-magnitude binary adder presented in [24] as a base. We extend its functionality to perform decimal addition/subtraction operations along with the original binary ones. Next, we briefly introduce the original binary adder and focus on our extensions. In the next subsection we will discuss the specific decimal functionalities of our design.

Let us assume $N 1$ and $N 2$ being two n-bit sign-magnitude numbers, such that $N 1=\left[N 1_{n-1} N 1_{n-2} \ldots N 1_{0}\right]$ and $N 2=$ [ $N 2_{n-1} N 2_{n-2} \ldots N 2_{0}$ ], with $N 1_{n-1}$ and $N 2_{n-1}$ used as sign bits of binary or BCD representations. Assuming absolute values, equation (1) is used to determine the exact operation, e.g. effective addition either subtraction. The logic one value of the signal Effective Addition (EAdd) indicates an effective addition operation, logic zero indicates that an effective subtraction is carried out.

$$
\begin{align*}
E A d d= & \left(\overline{N 1_{S}} \cdot \overline{N 2_{S}} \cdot \overline{A d d}\right)\left|\left(\overline{N 1_{S}} \cdot N 2_{S} \cdot A d d\right)\right| \\
& \left(N 1_{S} \cdot \overline{N 2_{S}} \cdot A d d\right) \mid\left(N 1_{S} \cdot N 2_{S} \cdot \overline{A d d}\right)  \tag{1}\\
= & \left(N 1_{S} \oplus N 2_{S} \oplus \overline{A d d}\right)
\end{align*}
$$

where the input $A d d=0$ invokes addition operation, otherwise subtraction is performed. Please note that we use " $\mid$ " to represent a logical $O R$ and " + " for addition. Also observe the use of modified sign bits in (1). This modification proposed here is to cope with the cases of unsigned and complement numbers as will be explained later. The two modified sign bits are computed as follows:

$$
\begin{align*}
& N 1_{S}=N 1_{n-1} \cdot \text { Type }  \tag{2}\\
& N 2_{S}=N 2_{n-1} \cdot \text { Type } \tag{3}
\end{align*}
$$

where Type $=0$ signal, masks both sign bits $N 1_{n-1}$ and $N 2_{n-1}$ in the case of unsigned and complement representations. The result of this will be that EAdd is equal to the
$\overline{A d d}$ signal making sure the final effective operation to be realized by the sign-magnitude adder is always correct. In addition, we should use $N 1_{n-1}$ and $N 2_{n-1}$ values in the result $(S U M)$ computation. How those values participate into the above operation for all possible cases is stated in Table 2.

Table 2. Adder setting up considerations

| Rep. | $E A d d$ | SUM | Type |
| :---: | :---: | :---: | :---: |
| S.Mag. | $N 1_{n-1}, N 2_{n-1}$ | $N 1_{n-1}=N 2_{n-1}=0$ | 1 |
| Uns. | $N 1_{n-1}=N 2_{n-1}=0$ | $N 1_{n-1}=N 2_{n-1}=0$ | 0 |
| Comp. | $N 1_{n-1}=N 2_{n-1}=0$ | $N 1_{n-1}, N 2_{n-1}$ | 0 |

The binary sign-magnitude addition is performed using the absolute values of the input addends as is stated by equation (4):

$$
\begin{equation*}
\sum=|N 1|+|N 2|^{*} \tag{4}
\end{equation*}
$$

where $|N 2|^{*}$ is equal to $|N 2|$ for effective addition and $|N 2|^{*}$ equals to $\overline{|N 2|}$ in case of effective substraction ${ }^{\text {ii }}$. In order to generate a correct sign-magnitude result, an additional correction step $\oplus \triangle$ is used. The final magnitude result becomes:

$$
\begin{equation*}
|S U M|=\sum_{k} \oplus \triangle \quad \forall 0<k<n-2 \tag{5}
\end{equation*}
$$

The $\triangle$ is computed as follows:

$$
\begin{equation*}
\triangle=\overline{C_{o}} \cdot \overline{E A d d} \tag{6}
\end{equation*}
$$

where the carry output $(\mathrm{Co})$ is equal to:

$$
\begin{align*}
C o= & G_{0}^{n-2}\left[|N 1|,|N 2|^{*}\right] \\
= & G_{0} \mid\left(P_{0} \cdot G_{1} \mid \ldots\left(P_{0} \cdot P_{1} \ldots \cdot P_{i-1} \cdot G_{i}\right.\right.  \tag{7}\\
& \left.|\ldots|\left(P_{0} \cdot P_{1} \ldots \cdot P_{i-3} \cdot G_{n-2}\right)\right)
\end{align*}
$$

where $G_{i}$ and $P_{i}$ are the generate and propagate signals of the single bit adders. Finally, the sign bit of the result is updated as shown in equation 8 (see [24] for details):

$$
\begin{equation*}
S U M_{n-1}=\left[N 1_{n-1} \oplus(\overline{C o} \cdot \overline{E A d d})\right] \cdot \overline{(S U M \equiv 0)} \tag{8}
\end{equation*}
$$

All of the different cases of the original adder are presented in the table shown in Figure 4.


Figure 4. Sign Magnitude Adder
${ }^{\text {ii }}$ EAdd signal is used to control the complement operation of N 2 .

For decimal operation we reuse the $E A d d$ and $A d d$ signals and introduce the two additional control signals (Bin and Type). Bin $=1$ causes a binary operation, otherwise a BCD operation is performed. The Type signal has the functionality as explained earlier.

### 3.1 Decimal Arithmetic Additions

In this section we describe in more details the specific additions to the original binary adder needed for decimal addition/subtraction operations. As already indicated, previous proposals use $\pm 6$ correction terms for the input operands and the result of the addition. Furthermore, when the subtrahend is greater than the minuend the result of the addition is in its complement representation and an extra cycle for result correction is required $[10,15,16]$. In our proposal the Co signal stated in equation (7) is used to detect the case when $N 2>N 1$ and decimal subtraction operations is performed. Its value in combination with the Decimal Carry signals determine the specific correction of the result, e.g. the additional ten's complement operation. A simple example of such a case is presented in Figure 5, which depicts all operations involved in a subtraction operation (when $N 2>N 1$ ).


Figure 5. Decimal subtraction: N2 $>$ N1
In the proposed adder depicted in Figure 6, the nine's complement computation for the subtrahend is performed using a "DigitWise-6" ( $D W$ ) hardwired logic as used in many previous designs [15]. The DW value $N D=N-6_{10}$ is obtained with the following equations that modify each bit of the BCD nibble as follows:
$N D_{(3)}=\overline{N 2_{(3)}} \cdot \overline{N 2_{(2)}}\left|\overline{N 2_{(3)}} \cdot \overline{N 2_{(1)}}\right| N 2_{(3)} \cdot N 2_{(2)} \cdot N 2_{(1)}$
$N D_{(2)}=N 2_{(2)} \oplus N 2_{(1)}$
$N D_{(1)}=\overline{N 2_{(1)}}$
$N D_{(0)}=N 2_{(0)}$
The digit carry logic $(D C)$ [5] signals for decimal operations are obtained as follows:

$$
\begin{equation*}
D C=X \mid Y \cdot C I \tag{10}
\end{equation*}
$$

where

$$
\begin{aligned}
X & =G_{3}\left|P_{3} \cdot P_{2}\right| P_{3} \cdot P_{1} \mid G_{2} \cdot P_{1} \\
Y & =P_{3}\left|G_{3}\right| P_{1} \cdot G_{1} \\
C I & =G_{1} \mid P_{1} \cdot C_{i n}
\end{aligned}
$$

All necessary correction cases for decimal data arithmetic, proposed here are summarized in Table 3. Our slightly more complex coder enables effective additionsubtraction characteristics for our universal adder. Please note that in the post-complement operation case, two necessary correction operations are required, both using $0110_{2}$ value (one on the subtrahend value and one on the correction of addition result). In our proposal, in contrast to all previous adders, a single addition with $1100_{2}$ value is used, achieving with this a real effective addition/subtraction capability.

Table 3. Decimal Correction Terms

| Operation | $D C$ |  | $\overline{D C}$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $N 1>N 2$ | $N 2 \geq N 1$ | $N 1>N 2$ | $N 2 \geq N 1$ |
| Addition | $0110_{2}$ | $0110_{2}$ | $0000_{2}$ | $0000_{2}$ |
| Subtraction | $0110_{2}$ | $1100_{2}$ | $0000_{2}$ | $0110_{2}$ |

Please note that when a binary operation is performed Bin $=1$ the decimal correction term is not needed. A value of $0000_{2}$ is used for any binary operation. The universal adder is set up with the aforementioned logic, a onedimensional (3/2)counter array, a Carry-propagate Adder and a set of XOR gates. The final organization is depicted in figure 6.


Figure 6. Universal adder micro-architecture

Note that the input $N 2^{*}$ for computing the digit carry logic is equal to $\overline{N 2}+10$ when processing decimal subtraction otherwise is equal to $N 2$. The multiplexor signal control for decimal subtraction or any addition $(D S)$ is computed by:

$$
\begin{equation*}
D S=\overline{A d d} \cdot \overline{B i n} \tag{12}
\end{equation*}
$$

The final complement operation is controlled by equation (13) which is a modified version of equation (6):

$$
\begin{equation*}
\triangle=\left(\overline{C_{o}} \cdot \overline{E A d d}\right) \mid\left(C_{o} \cdot \overline{A d d} \cdot \overline{\operatorname{Bin}}\right) \tag{13}
\end{equation*}
$$

The rationale behind the design of the universal adder is based on the following observations and considerations:

- EAdd signal controls one's complement operation of the subtrahend in binary and decimal operations.
- The $\triangle$ logic (see equation (13)) corrects the final result in binary and decimal operations.
- We need a (3/2)counter row to selectively add the necessary correction terms (correction coder) as indicated by Table 3.
- The hardware reuse is fundamental when a partial reconfigurable hardware units are designed for reconfigurable processor scenarios.

When our design is mapped on partially reconfigurable hardware platforms, the following modules (shown in dark in Figure 6) can be reused in both modes: the Carry-propagate Adder, the EAdd logic, the two XOR logic blocks (at the input and output of the unit) and the Co logic. These "permanent" blocks can be configured at the beginning when the processor is set up. In order to provide the BCD functionality the remaining (shown as not colored in Figure 6) modules can be configured on demand. Such dynamic partial reconfiguration will diminish the penalty due to reconfiguration latencies. A preliminary estimation [6] indicates that the "permanent" blocks account for approximately $40 \%$ of the total hardware resources implementing the operations when targeting Xilinx Virtex 4 devices.

Finally, the fundamental decimal addition and subtraction operations of our proposal can be summarized with the following equations ${ }^{\text {iii }}$ :

## Decimal Addition

$$
\begin{aligned}
& \text { Sum }=N 1+N 2+0110_{2} \forall D C=1 \\
& \text { Sum }=N 1+N 2+0000_{2} \forall D C=0
\end{aligned}
$$

## Decimal Subtraction

$$
\begin{aligned}
& S u m=N 1+D W(\overline{N 2})+0110_{2} \forall D C=1 \wedge C o=0 \\
& S u m=N 1+D W(\overline{N 2})+0000_{2} \forall D C=0 \wedge C o=0 \\
& S u m=N 1+D W(\overline{N 2})+1100_{2} \forall D C=1 \wedge C o=1 \\
& S u m=N 1+D W(\overline{N 2})+0110_{2} \forall D C=0 \wedge C o=1
\end{aligned}
$$

[^1]
## 4 Experimental Results Analysis

The proposed Universal Adder was implemented using VHDL, synthesized, functionally tested, and evaluated using the ISE 8.1i Xilinx design tools [27] targeting 4vfx60ff672-12 VIRTEX 4 FPGA device. Furthermore, the designs proposed by Fischer [10], Hwang [17], Busaba [5] and Haller [16] were also implemented and synthesized using the same methodology. Table 4, summarizes the latency and hardware utilization results for all of the considered designs.

Table 4. Latency\&Area - Adder comparison

| 32-bit wide units | Latency (ns) |  |  | Resources used |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Logic | Wire | Total | Slices | LUTs |
| Our proposal | 7.3 | 4.7 | 12.1 | 256 | 495 |
| Fischer [10] | 5.5 | 4.8 | 10.3 | 123 | 233 |
| Hwang [17] | 5.9 | 4.6 | 10.5 | 82 | 158 |
| Busaba [5] | 5.3 | 5.6 | 10.9 | 242 | 466 |
| Haller [16] | 5.5 | 4.5 | 10.0 | 305 | 584 |

Please note that all of the units proposed before require an additional cycle to produce the correct result in cases when $N 2>N 1$ for the BCD subtraction operations (see Figure 5). To deal with this problem additional hardware unit or software fix is required. In the Fischer proposal the output stage can possible be modified for effective addition/subtraction, however, the custom made "editing logic", is expected to become very complex. In addition, none of the above proposals presented how they will detect the $N 2>N 1$ (BCD) situation. Our design does not require such additional effort or resources with some increase in latency and area.

In terms of latency the "best" design is the one proposed by Haller. Compared to it, our proposal is $21 \%$ slower, but uses $15 \%$ to $16 \%$ less hardware resources (LUTs and Slices). When considering area, Hwang's proposal ranks best, but has the deficiency as described above. In general, our design is between $11 \%$ and $21 \%$ slower than the others. Assuming a design with a latency of 10 ns , e.g. Haller, (worse case for our design) and a detection of $N 2>N 1$ (BCD) "for free", the average execution time is going to be similar to ours (12ns) when $20 \%$ of the operations require the compensation cycle as introduced earlier.

When absolute performance is considered, our design achieves 82.6 MOPS ( $1 / 12 \mathrm{~ns}$ ). Please note that this will be significantly improved when a pipelined version of the design is considered. The latency of the Carry-propagate Adder is expected to de determine the operating frequency in such a case.

## 5 Conclusions

This paper provides the details for a novel adder/subtracter arithmetic unit that combines Binary
and Binary Code Decimal (BCD) operations in a single structure. The unit is able to perform effective additionsubtraction operations on unsigned, sign-magnitude, and various complement representations. Our proposal can be implemented in ASIC as a run time configurable unit as well as in reconfigurable technology as a run-time reconfigurable engine. Under the assumption that significant part of the hardware in the proposed structure is shared by the different operations reconfigurable platforms with partial reconfiguration become an interesting target. The proposed unit and several widely used adder organizations were synthesized for 4vfx60ff672-12 Xilinx Virtex 4 FPGA for comparison. Our design achieves a throughput of 82.6 MOPS with similar area x time (AxT) product when compared to the other proposals, using only $2 \%$ of the available resources of the targeted FPGA.

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[^0]:    ${ }^{i}$ In the context of this article, we assume operands and results to be in unsigned, sign-magnitude or complement (two's complement for binary and ten's complement for decimal) notations.

[^1]:    ${ }^{\text {iii }}$ Please note that the decimal Sum value presented in those equations requires the one complemented operation as was stated in (5). Where the $\Delta$ control signal (13) is used to obtain the final $S U M$ result

